Navier-Stokes Solutions to Ship-Ship Interaction

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PASSING VESSEL ISSUES WORKSHOP

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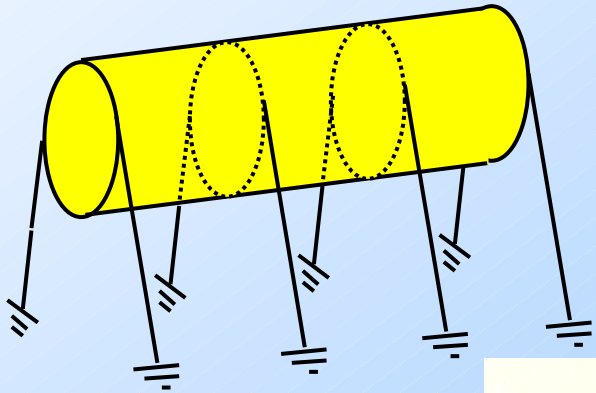
Objective

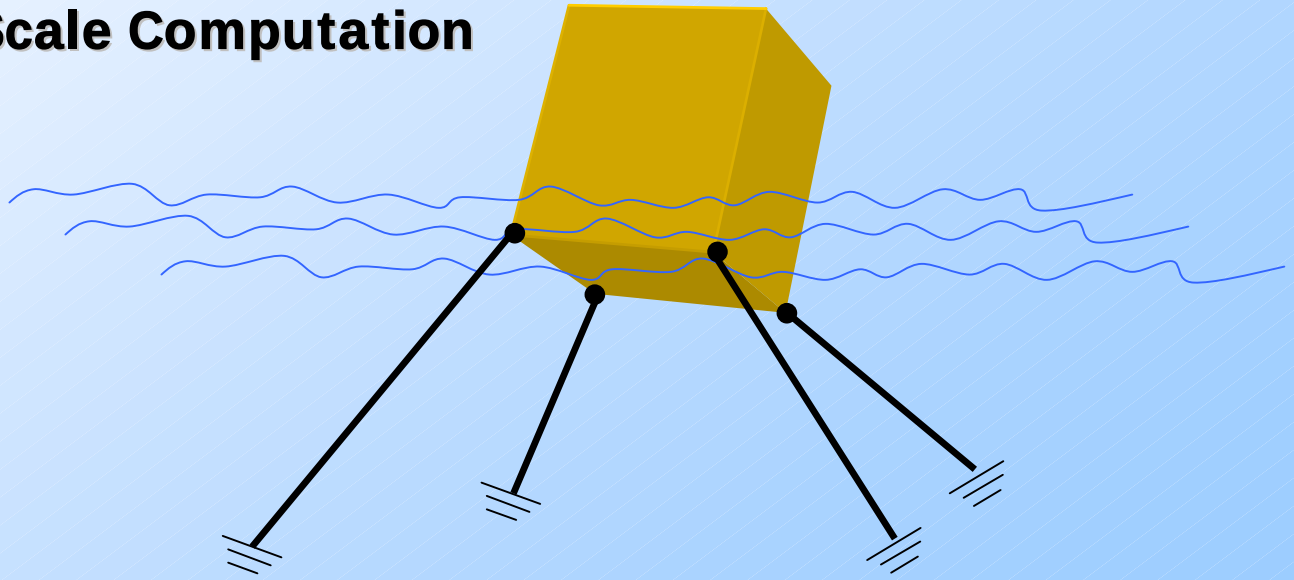
Develop HPC Fluid-Structure Interaction Technology for Large Scale Simulation of Free-Surface and Fluid-Structure Interaction Applications



- Mooring Forces
- Wave Formation
- Coastal Erosion
- Wave Impact
- Ship Hydrodynamics
- Floatation
- Sloshing
- Channel Flow

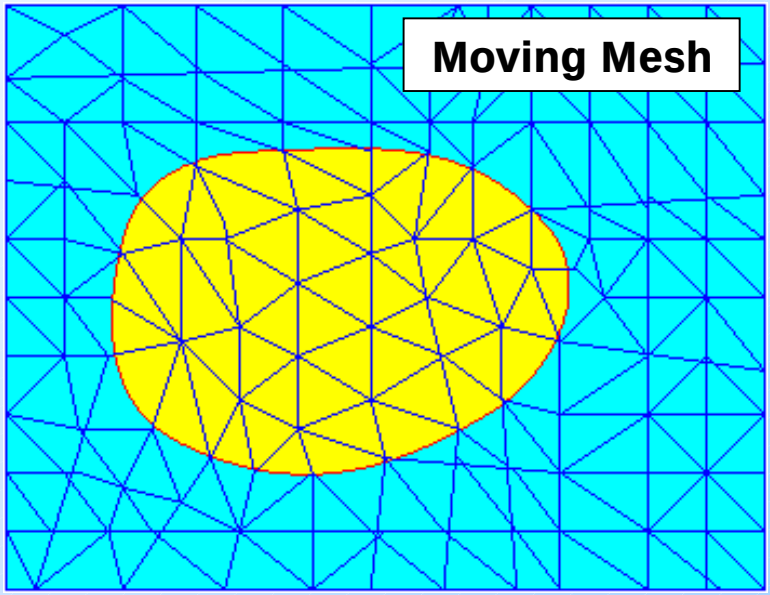
Challenges

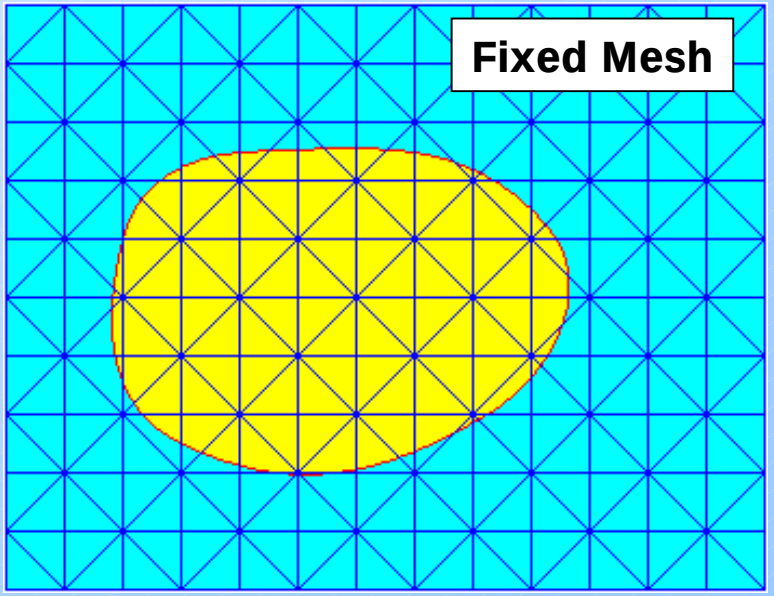
- Nonlinear Navier-Stokes Equations (u,v,w,p)
- Discontinuous Density and Viscosity
- Location of the Free-Surface is an unknown
- Moving Boundaries
- Nonlinear Rigid Body Dynamics, 6DOF
- Moving Mesh Mechanism
- Nonlinear Elasticity Equations for Hawsers and Structures
- Large- Scale Computation



Solution Technique

- Moving Mesh Methods (ALE, ST)
  - Accurate
  - Coarse Mesh Computation
  - Small Deformation
  - Mesh Moving
  - Mesh Distortion / Remeshing
  - Boundary Layer
- Fixed Mesh Methods (VOF)
  - No Mesh Moving
  - No Remeshing
  - Flexible
  - Large Deformation
  - Additional Equation
  - Refined Mesh





Governing Equations: Navier-Stokes Equations For Both Moving and Fixed Meshes

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}-\mathbf{g}\right)-\nabla\cdot\sigma=0$$

$$\nabla\cdot\mathbf{u}=0$$

$$\sigma=-p\mathbf{I}+2\mu\varepsilon(u)$$

$$\varepsilon=\frac{1}{2}(\nabla\mathbf{u}+\nabla\mathbf{u}^{T})$$

Governing Equations: Navier-Stokes Equations

Initial Conditions:

$$\mathbf{u} = \mathbf{u}_{IC} \qquad p = 0 \quad \text{or} \quad p = \rho g z$$

Boundary Conditions:

Inflow: $\mathbf{u} = \mathbf{u}_g \qquad \text{on } \Gamma_g$

Outflow: $\mathbf{n}.\sigma = \mathbf{h} \qquad \text{on } \Gamma_h$

Note: No Boundary Condition for Pressure

Governing Equations: Navier-Stokes Equations

Approximate Boundary Conditions:

$$\sigma \approx -p\mathbf{I} \qquad \text{Either} \quad \mu \approx 0 \quad \text{or} \quad \nabla \mathbf{u} \approx \mathbf{0}$$

Outflow: $\mathbf{n}.\sigma \approx -p\mathbf{n} \approx \mathbf{h} \quad \text{on } \Gamma_h$

Usually: $\mathbf{h} \approx (0, 0, 0) \quad \Rightarrow \quad p \approx 0$

Hydrostatic Outflow: $p \approx \rho g z \quad \Rightarrow \quad \mathbf{h} \approx (0, 0, -\rho g z)$

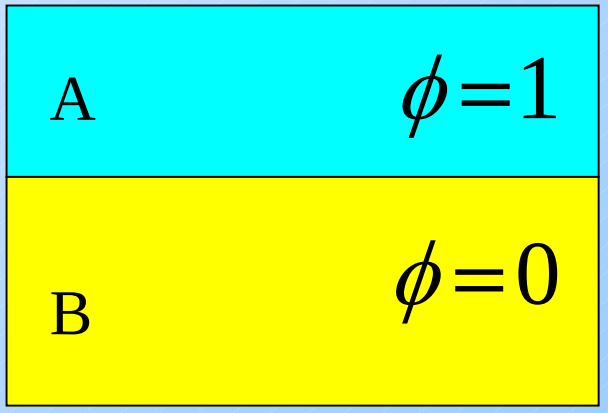
Fixed Meshes are Prefered for Flexibility

Governing Equations: Interface Function Equation

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0$$

$$\rho = \phi \rho_A + (1 - \phi) \rho_B$$

$$\mu = \phi \mu_A + (1 - \phi) \mu_B$$



Stabilized Finite Element Formulations

$$\int_\Omega \mathbf{w}^h \cdot \rho\left(\frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{u}^h - \mathbf{g}\right) d\Omega + \int_\Omega \boldsymbol{\varepsilon}(\mathbf{w}^h) : \boldsymbol{\sigma}(p^h, \mathbf{u}^h) d\Omega$$

$$+ \int_\Omega q_p^h \nabla \cdot \mathbf{u}^h d\Omega + \sum_{e=1}^{ne} \int_{\Omega^e} \tau_c \nabla \cdot \mathbf{w}^h \rho \nabla \cdot \mathbf{u}^h d\Omega$$

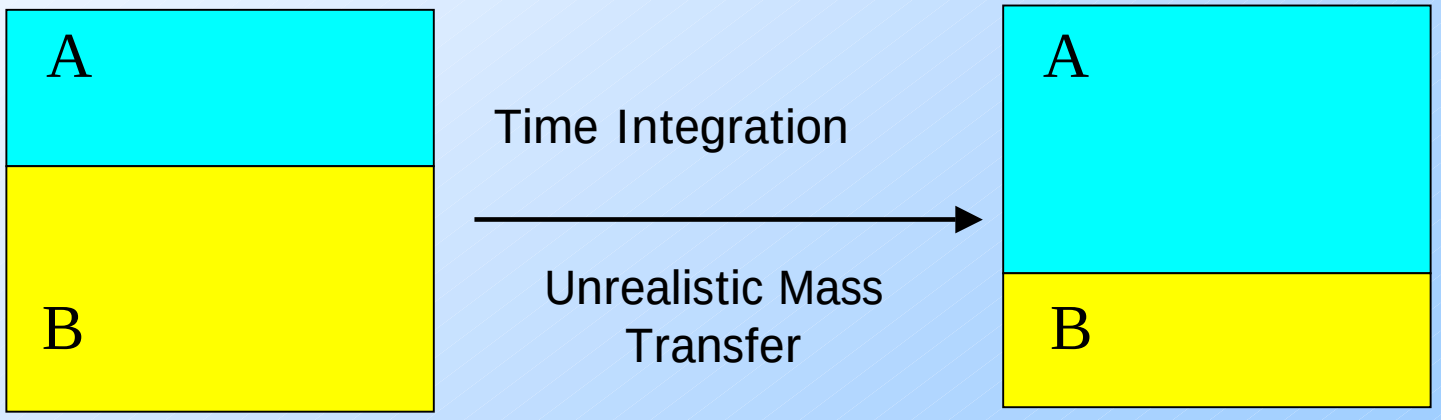
$$+ \sum_{e=1}^{ne} \int_{\Omega^e} \frac{\tau_m}{\rho} \left[\mathbf{u}^h \cdot \nabla \mathbf{w}^h - \nabla \cdot \boldsymbol{\sigma}(q_p^h, \mathbf{w}^h)\right] \cdot$$

$$\left[\rho\left(\frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{u}^h - \mathbf{g}\right) - \nabla \cdot \boldsymbol{\sigma}(p^h, \mathbf{u}^h)\right] d\Omega = \int_{\Gamma_{h_u}} \mathbf{w}^h . \mathbf{h}\, d\Gamma$$

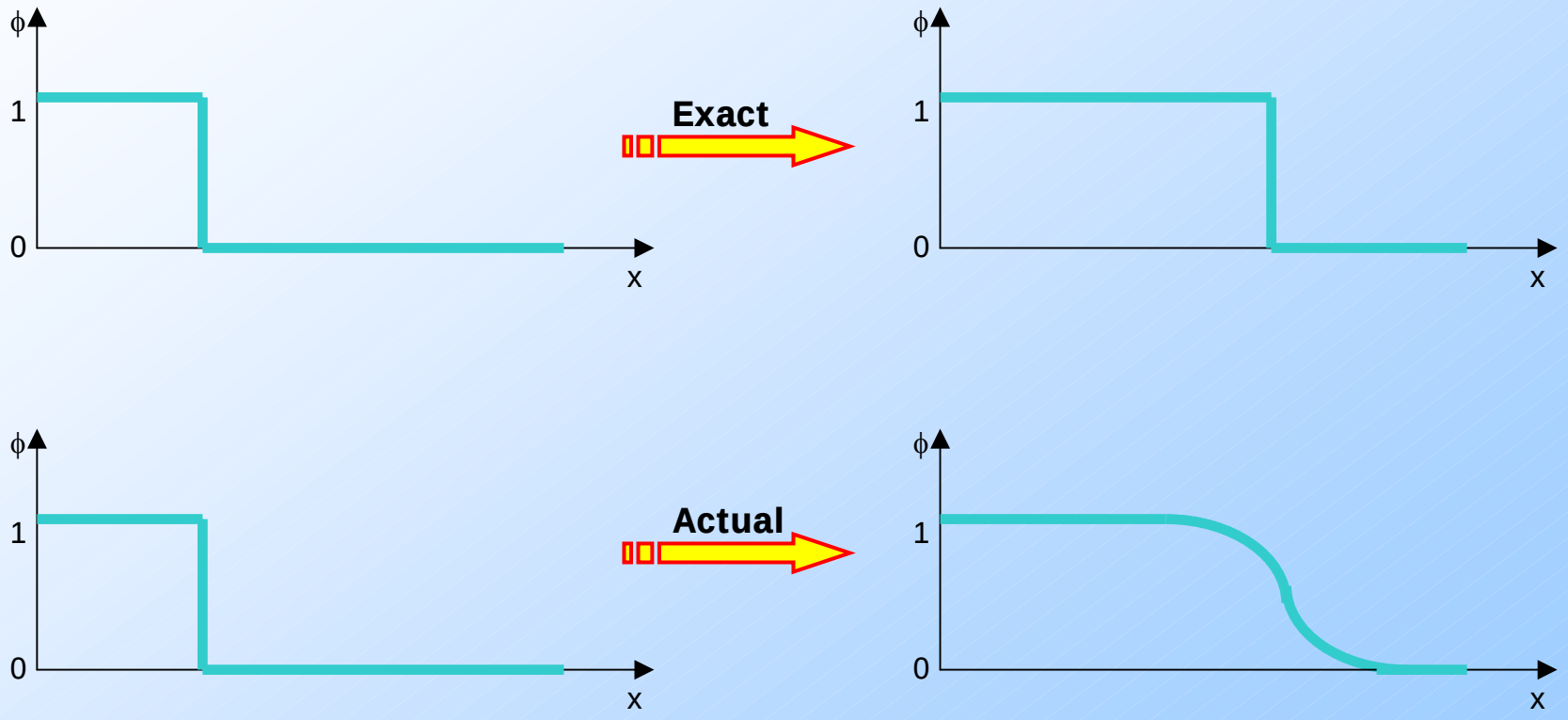
$$\int_\Omega \psi^h \left(\frac{\partial \phi^h}{\partial t} + \mathbf{u}^h \cdot \nabla \phi^h\right) d\Omega + \sum_{e=1}^{ne} \int_{\Omega^e} \nu \nabla \psi^h \cdot \nabla \phi^h d\Omega = 0$$

Fixed-Mesh Method Problem!

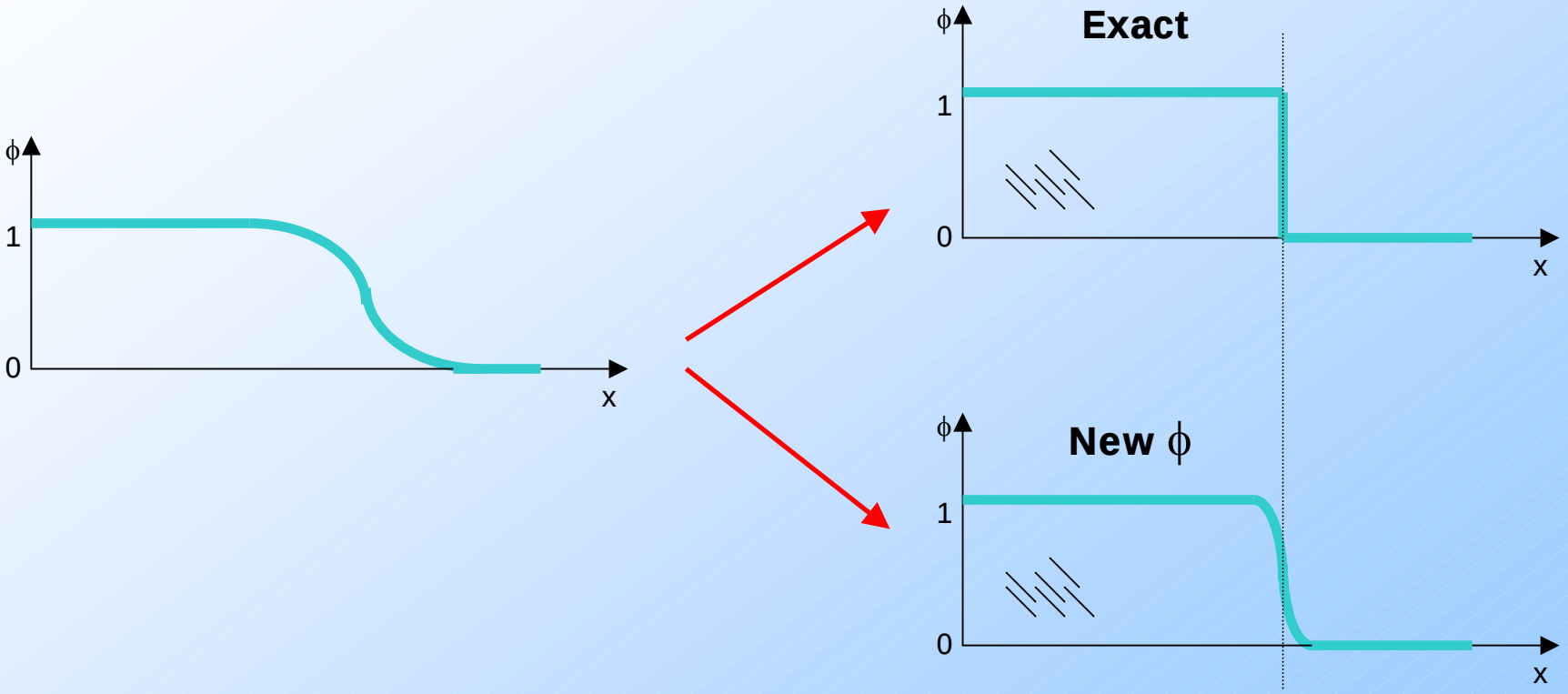
- Numerical Errors Results in Fictitious Mass Transfer
  - Artificial Diffusion Stabilization Error
  - Spatial Discretization Error



Interface Sharpening and Global Mass Conservation Algorithm



Interface Sharpening and Global Mass Conservation (IS-GMC)



IS-GMC Technique

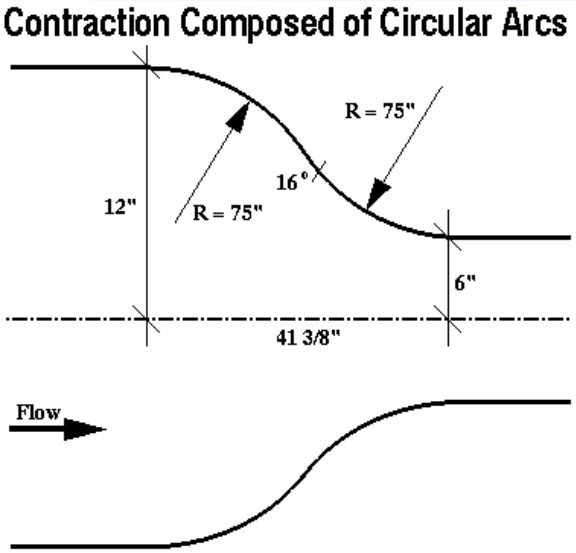
$$\phi_{new} = \begin{cases} \beta\left(\dfrac{\phi}{\beta}\right)^{\alpha}, & 0 \le \phi \le \beta \\ 1-(1-\beta)\left(\dfrac{1-\phi}{1-\beta}\right)^{\alpha}, & \beta \le \phi \le 1 \end{cases}$$

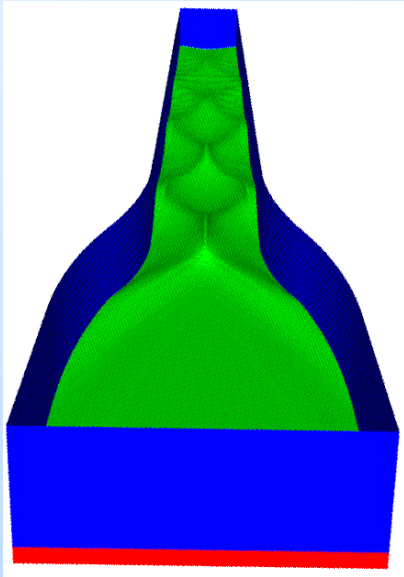
- **To find b, we satisfy global mass conservation:**

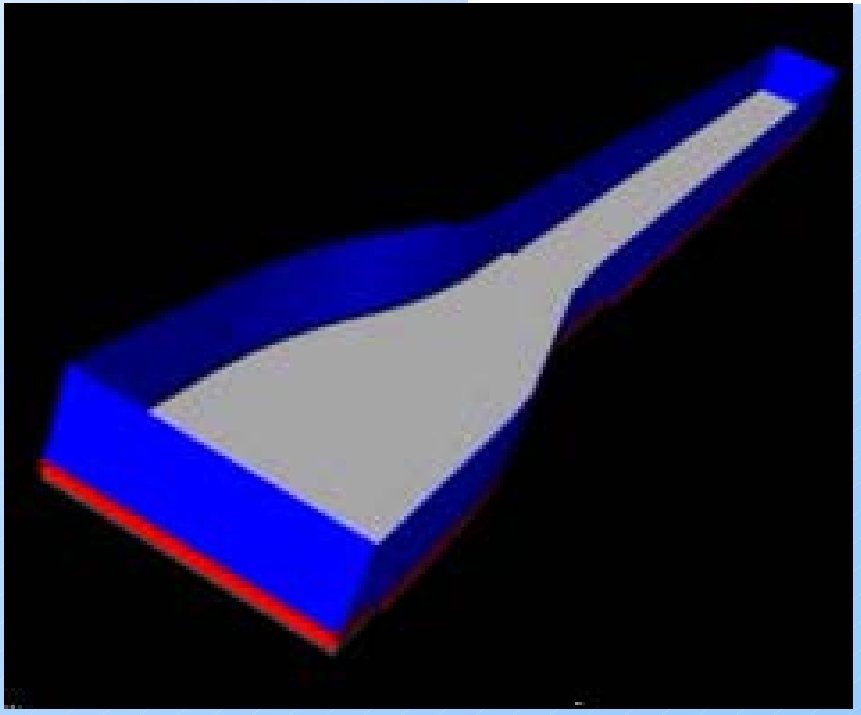
$$\rho_A \int_{\Omega} \phi_{new} d\Omega = m_A + \rho_A \int_t \int_{\Gamma} \phi \mathbf{u.n}\, d\Gamma dt$$

$$M\beta^{(1-\alpha)} + N(1-\beta)^{(1-\alpha)} = R$$

Accuracy of the Method in 3D:
Contraction Channel

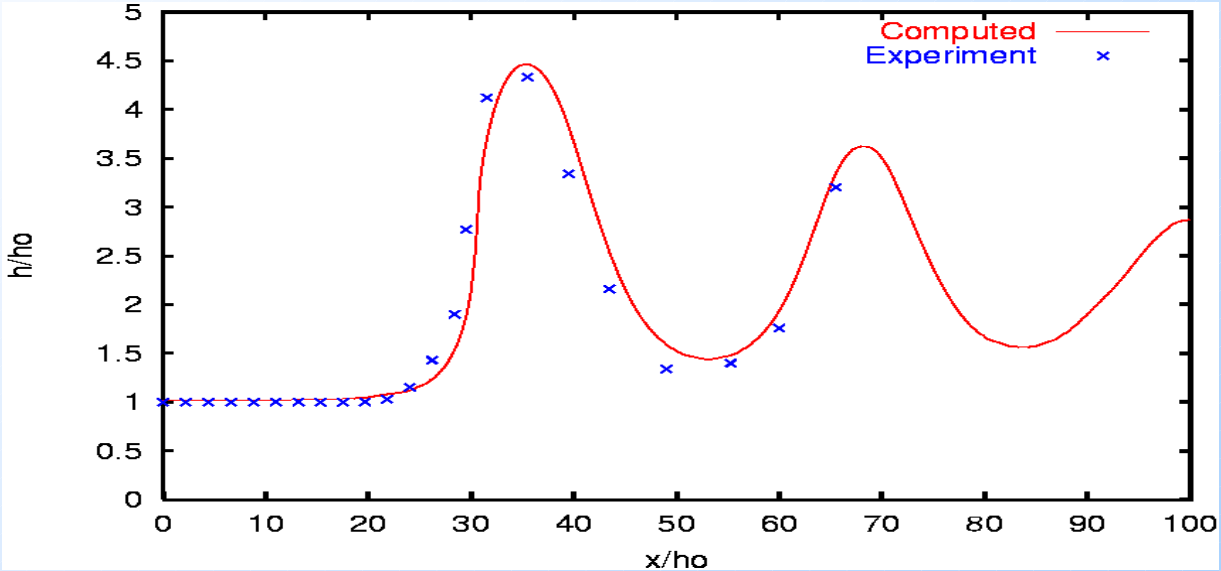




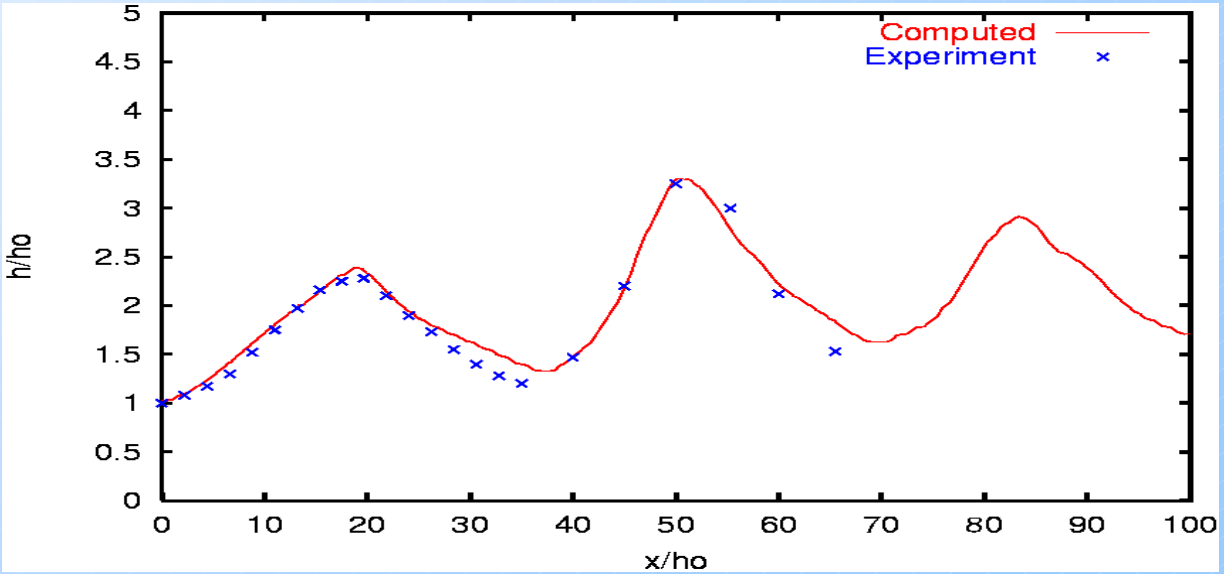


- 401x61x111
- Fr = 4
- Equations: 13 Million

Along the Center

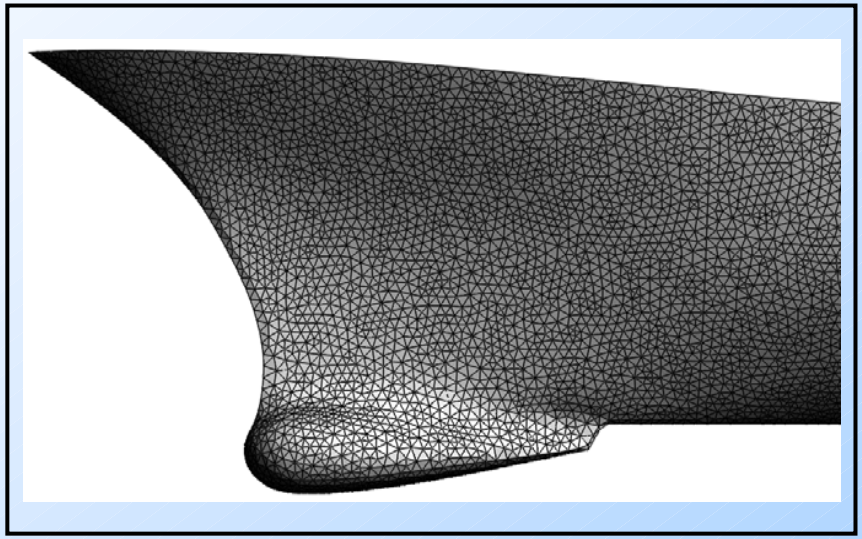


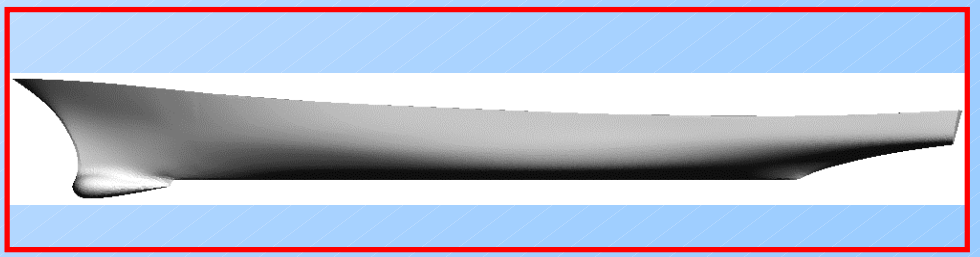
Along the Wall



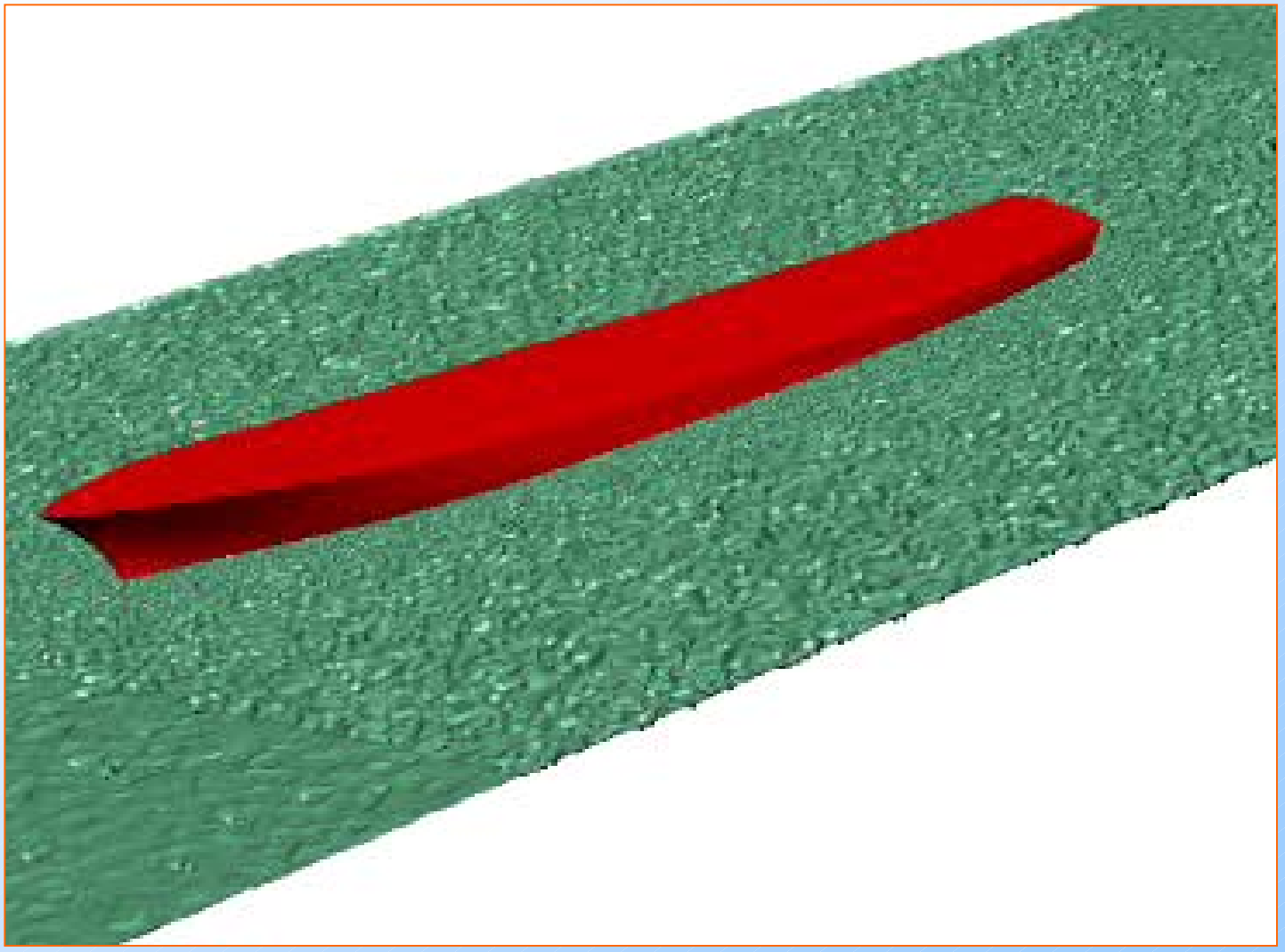
Ship Hydrodynamics – DTMB 5415

- Code Verification
- Experimental Data Exist
- Fr=0.28 and 0.41
- Mesh: 25 Million Element

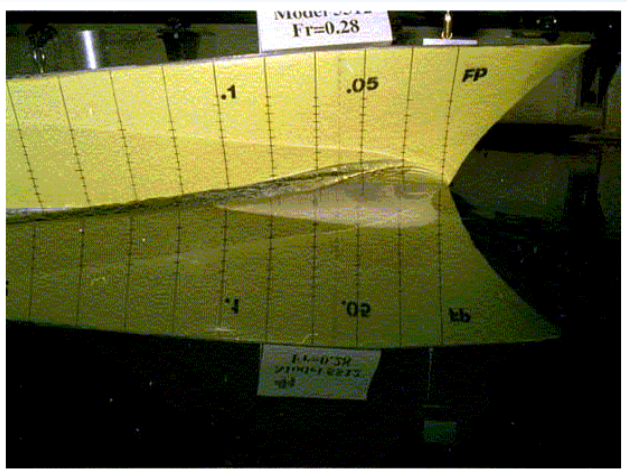


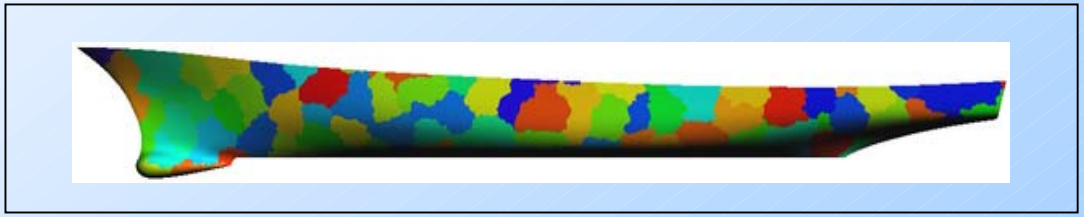


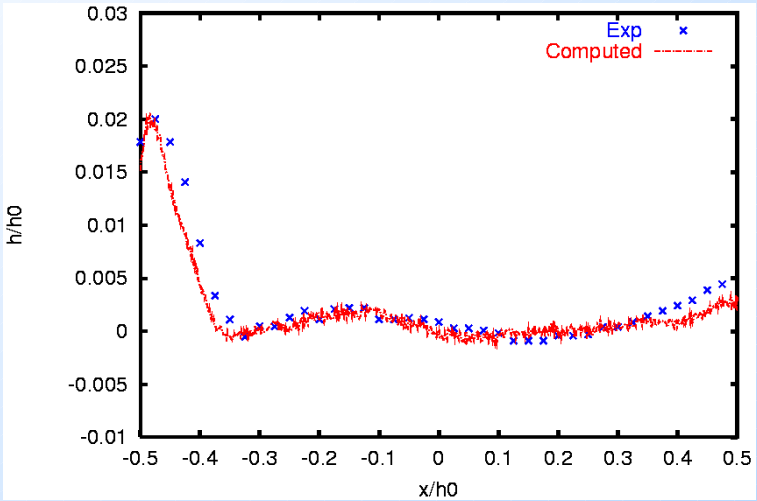
DTMB 5415: Animation

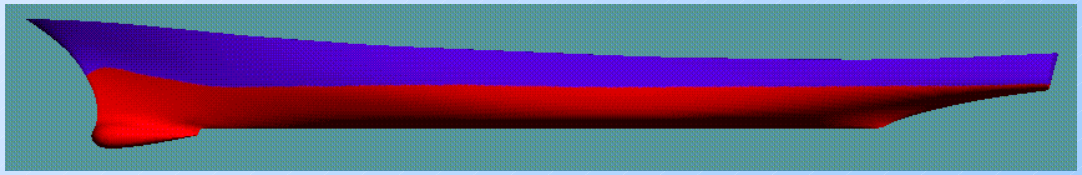


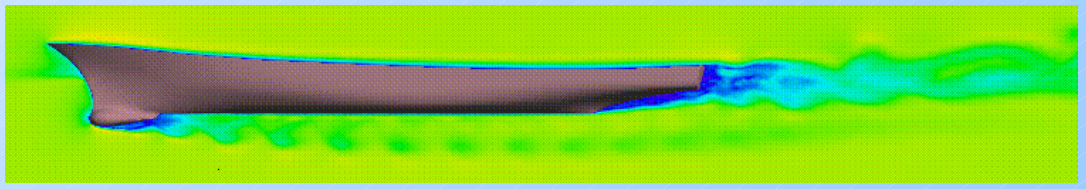
DTMB 5415: Fr=0.28



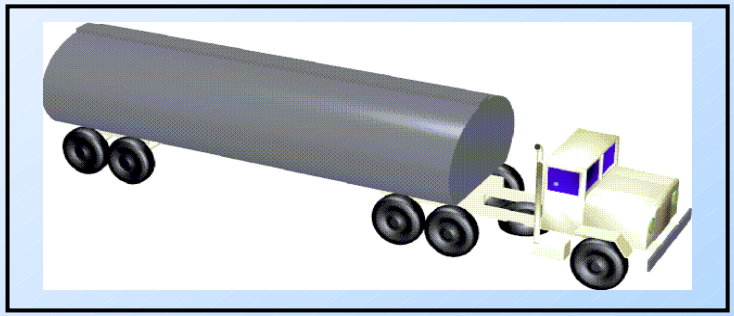


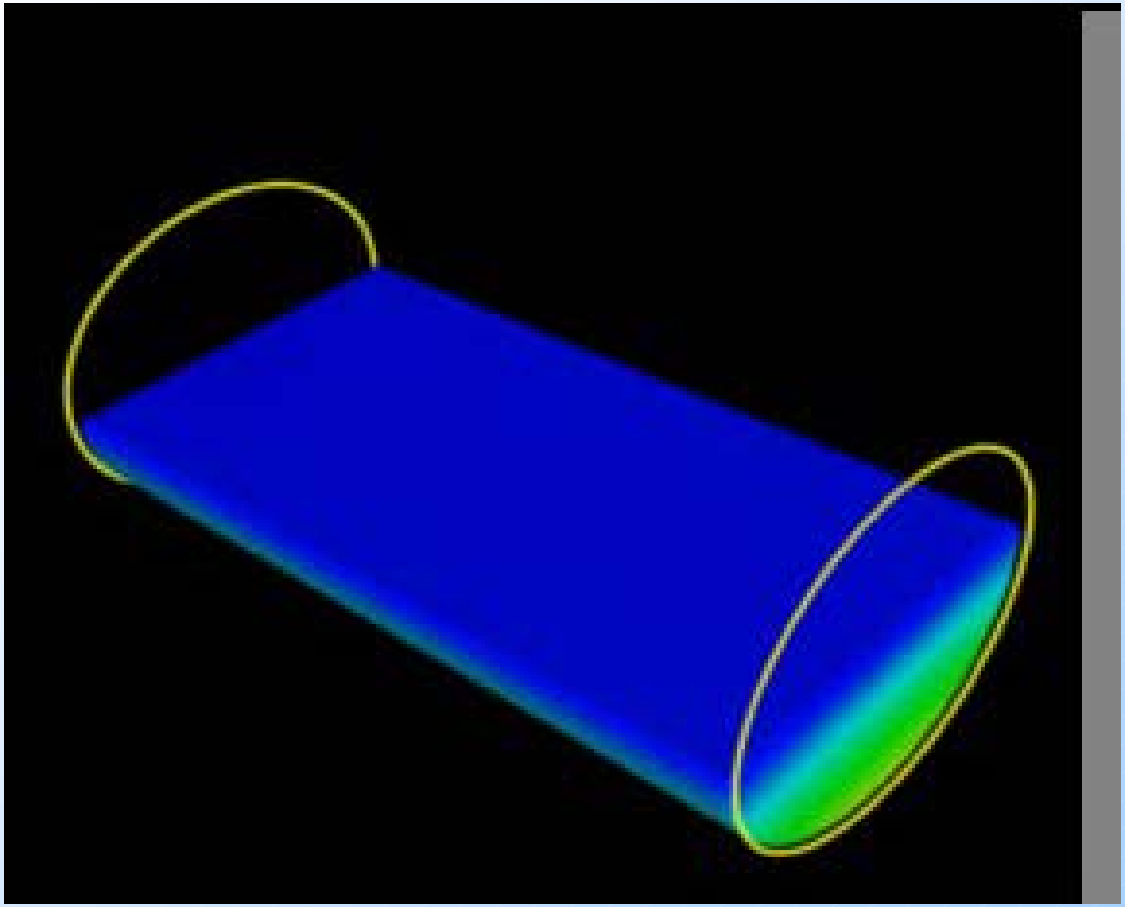


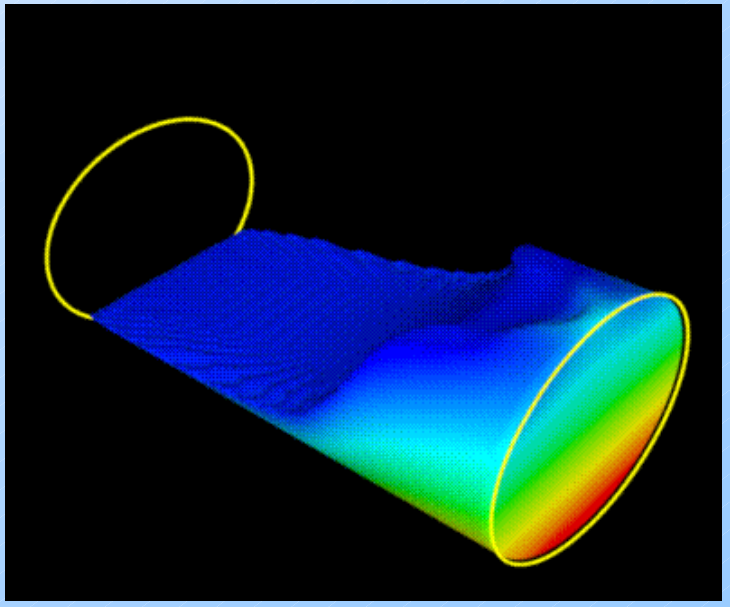




3D Application:
Sloshing In Tanker Truck



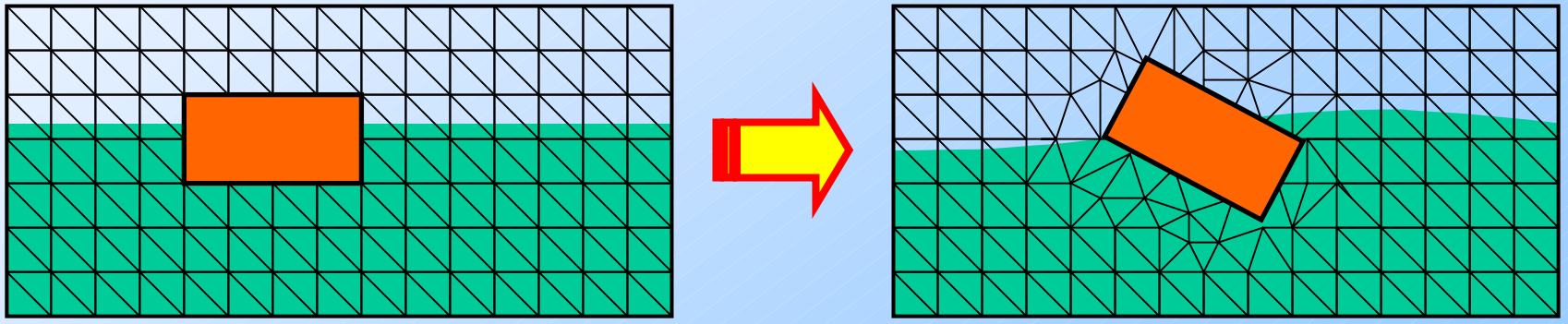




Free-Surface Flow with Moving Boundaries

- Fixed-Mesh Technique to Handling the Free-Surface
- Moving Mesh Technique to Handle the Motion of Physical Boundaries

Combination Technique



Governing Equations: Navier-Stokes Equations

$$\rho\left[\left.\frac{\partial \mathbf{u}}{\partial t}\right|_{\xi} + \left(\mathbf{u} - \mathbf{u}_{\text{mesh}}\right)\cdot\nabla\mathbf{u} - \mathbf{g}\right] - \nabla\cdot\sigma = 0$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\sigma = -\mathrm{p}\,\mathbf{I} + 2\mu\varepsilon(u)$$

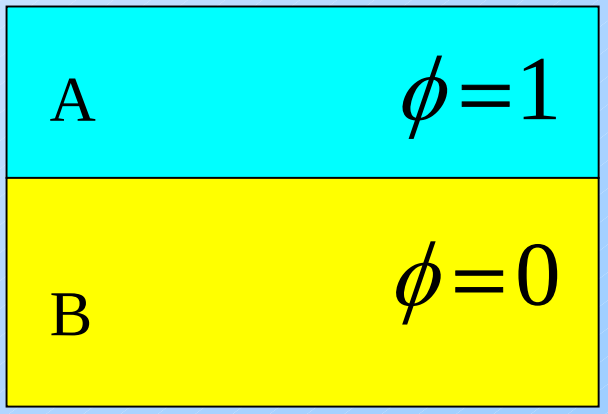
$$\varepsilon = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}})$$

Governing Equations: Interface Function Equation

$$\left.\frac{\partial \phi}{\partial t}\right|_{\xi} + \left(\mathbf{u} - \mathbf{u}_{\text{mesh}}\right) \cdot \nabla \phi = 0$$

$$\rho = \phi \rho_A + (1 - \phi) \rho_B$$

$$\mu = \phi \mu_A + (1 - \phi) \mu_B$$



Governing Equations: Rigid Body Dynamics

$$\mathbf{F}_X - m\,\mathbf{g}_X = m\,\mathbf{a}_X$$

$$\mathbf{M}_Y = \mathbf{J}_Y\,\boldsymbol{\alpha}_Y$$

$$\mathbf{u}_P^{n+1} = \mathbf{v}^{n+1} + \boldsymbol{\omega}^{n+1} \times (\mathbf{r}_P^n - \mathbf{C}^n)$$

$$\mathbf{d}_P^{n+1} = \Delta\mathbf{C}^{n+1} + \Delta\boldsymbol{\theta}^{n+1} \times (\mathbf{r}_P^n - \mathbf{C}^n)$$

$$\mathbf{a}^{n+1} = \frac{\mathbf{F}^{n+1}}{m} - \mathbf{g}$$

$$\mathbf{v}^{n+1} = \mathbf{v}^n + \Delta t\left(\frac{\mathbf{a}^{n+1} + \mathbf{a}^{n+1}}{2}\right)$$

$$\Delta\mathbf{C}^{n+1} = \Delta t\left(\frac{\mathbf{v}^{n+1} + \mathbf{v}^{n+1}}{2}\right)$$

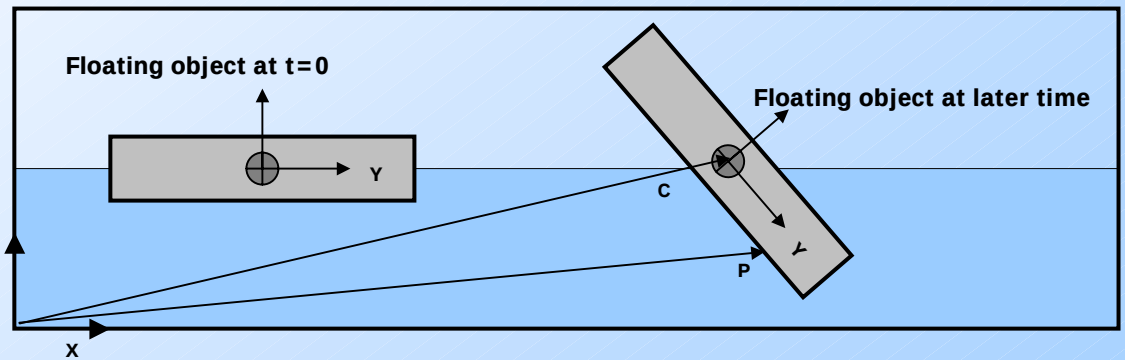
$$\boldsymbol{\alpha}^{n+1} = \mathbf{R}^{n+1}\mathbf{J}_Y^{-1}\mathbf{R}^{n+1^T}\mathbf{M}$$

$$\boldsymbol{\omega}^{n+1} = \boldsymbol{\omega}^n + \Delta t\left(\frac{\boldsymbol{\alpha}^{n+1} + \boldsymbol{\alpha}^n}{2}\right)$$

$$\Delta\boldsymbol{\theta}^{n+1} = \Delta t\left(\frac{\boldsymbol{\omega}^{n+1} + \boldsymbol{\omega}^n}{2}\right)$$

Compute $[\mathbf{S}]$ based on $\Delta\theta_1^{n+1}, \Delta\theta_2^{n+1}, \Delta\theta_3^{n+1}$

$$\mathbf{R}^{n+1} = \mathbf{S}\mathbf{R}^n$$

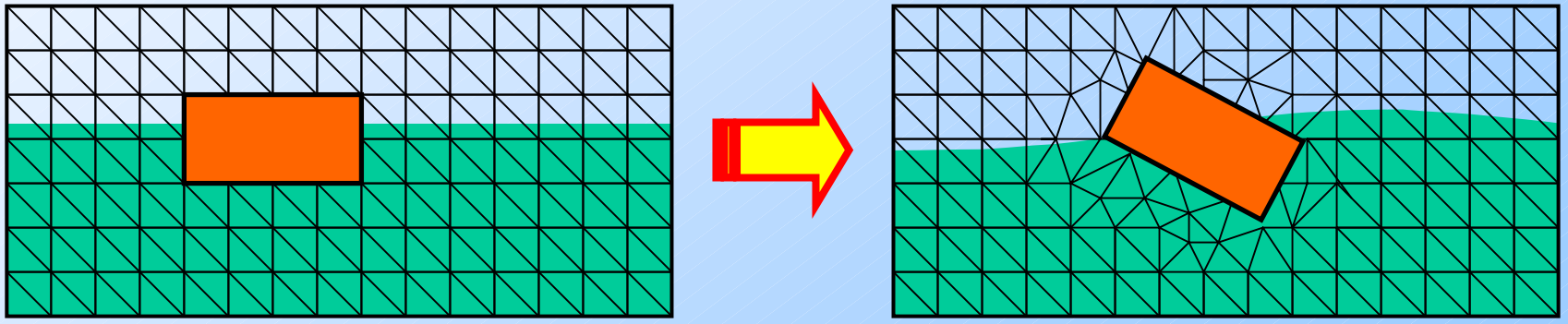


Governing Equations:

Linear Elasticity Equations for Mesh-Moving

$$\nabla . \left[\lambda_1 (\nabla . \mathbf{d}) \mathbf{I} + 2\lambda_2 \nabla \boldsymbol{\kappa}(\mathbf{d}) \right] = \mathbf{0}$$

$$\boldsymbol{\kappa} = \frac{1}{2} (\nabla \mathbf{d} + \nabla \mathbf{d}^{\mathrm{T}})$$



Finite Element Formulation

$$\int_\Omega \mathbf{w}^h \cdot \rho[\frac{\partial \mathbf{u}^h}{\partial t} + (\mathbf{u}^h - \mathbf{u}^h_{\text{mesh}}) \cdot \nabla \mathbf{u}^h - \mathbf{g}] d\Omega + \int_\Omega \boldsymbol{\varepsilon}(\mathbf{w}^h) : \boldsymbol{\sigma}(p^h, \mathbf{u}^h) d\Omega$$

$$+ \int_\Omega q^h_p \nabla \cdot \mathbf{u}^h d\Omega + \sum_{e=1}^{ne} \int_{\Omega^e} \tau_c \nabla \cdot \mathbf{w}^h \rho \nabla \cdot \mathbf{u}^h d\Omega$$

$$+ \sum_{e=1}^{ne} \int_{\Omega^e} \frac{\tau_m}{\rho} \left[(\mathbf{u}^h - \mathbf{u}^h_{\text{mesh}}) \cdot \nabla \mathbf{w}^h - \nabla \cdot \boldsymbol{\sigma}(q^h_p, \mathbf{w}^h) \right] \cdot$$

$$\left[\rho[\frac{\partial \mathbf{u}^h}{\partial t} + (\mathbf{u}^h - \mathbf{u}^h_{\text{mesh}}) \cdot \nabla \mathbf{u}^h - \mathbf{g}] - \nabla \cdot \boldsymbol{\sigma}(p^h, \mathbf{u}^h) \right] d\Omega = \int_{\Gamma_{h_u}} \mathbf{w}^h . \mathbf{h} \, d\Gamma$$

$$\int_\Omega \psi^h \left[\frac{\partial \phi^h}{\partial t} + (\mathbf{u}^h - \mathbf{u}^h_{\text{mesh}}) \cdot \nabla \phi^h \right] d\Omega + \sum_{e=1}^{ne} \int_{\Omega^e} \tau_i \nabla \psi^h \cdot \nabla \phi^h d\Omega = 0$$

$$\int_\Omega \boldsymbol{\kappa}(\boldsymbol{\pi}^h) : \left(\lambda_1 (\nabla . \mathbf{d}^h) \mathbf{I} + 2\lambda_2 \nabla \boldsymbol{\kappa}(\mathbf{d}^h) \right) d\Omega = \int_{\Gamma_{h_d}} \boldsymbol{\pi}^h . \mathbf{h}_d \, d\Gamma$$

Discretization:

$$w_{\mathrm{i}} = \sum_{\mathrm{A}=1,\,\mathrm{nn}} N_{\mathrm{A}} C_{\mathrm{Ai}}$$

$$w_1 = u_1 \quad w_2 = u_2 \quad w_3 = u_3 \quad w_4 = q$$

$$w_5 = \psi \quad w_6 = \pi_1 \quad w_7 = \pi_2 \quad w_8 = \pi_3$$

$$s_{\mathrm{i}} = \sum_{\mathrm{A}=1,\,\mathrm{nn}} N_{\mathrm{A}} u_{\mathrm{Ai}}^{p} \qquad \mathrm{i} = 1, 2, 3$$

$$s_4 = \sum_{\mathrm{A}=1,\,\mathrm{nn}} N_{\mathrm{A}} p_{\mathrm{A}}^{p}$$

$$s_5 = \sum_{\mathrm{A}=1,\,\mathrm{nn}} N_{\mathrm{A}} \phi_{\mathrm{A}}^{p}$$

$$s_{\mathrm{j}} = \sum_{\mathrm{A}=1,\,\mathrm{nn}} N_{\mathrm{A}} d_{\mathrm{Aj}}^{p} \qquad \mathrm{j} = 6, 7, 8$$

Iterative Solution Strategy

$$\mathbf{F}(\mathbf{s},\mathbf{s}) = \mathbf{L}$$

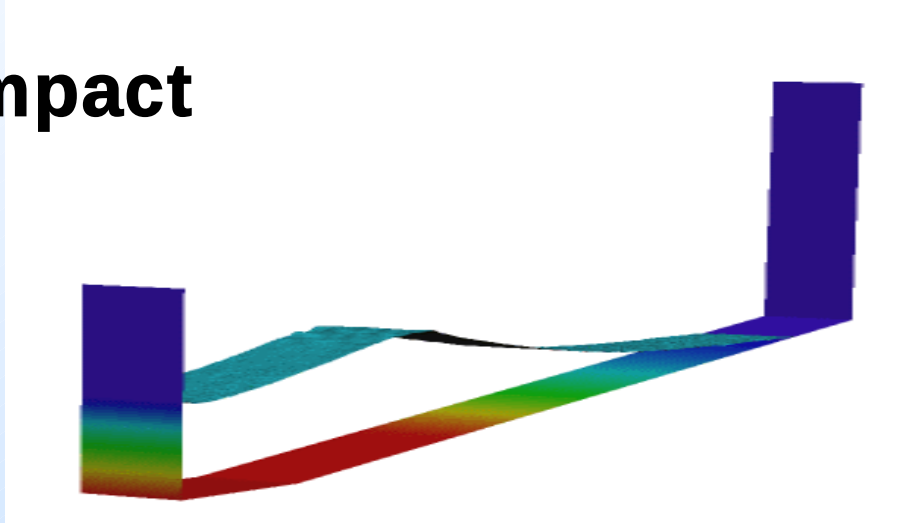
$$\mathbf{M}\,\Delta\mathbf{s}^{\tau+1} + \mathbf{N}\,\Delta\mathbf{s}^{\tau+1} = \mathbf{L} - (\mathbf{F}^{\tau})$$

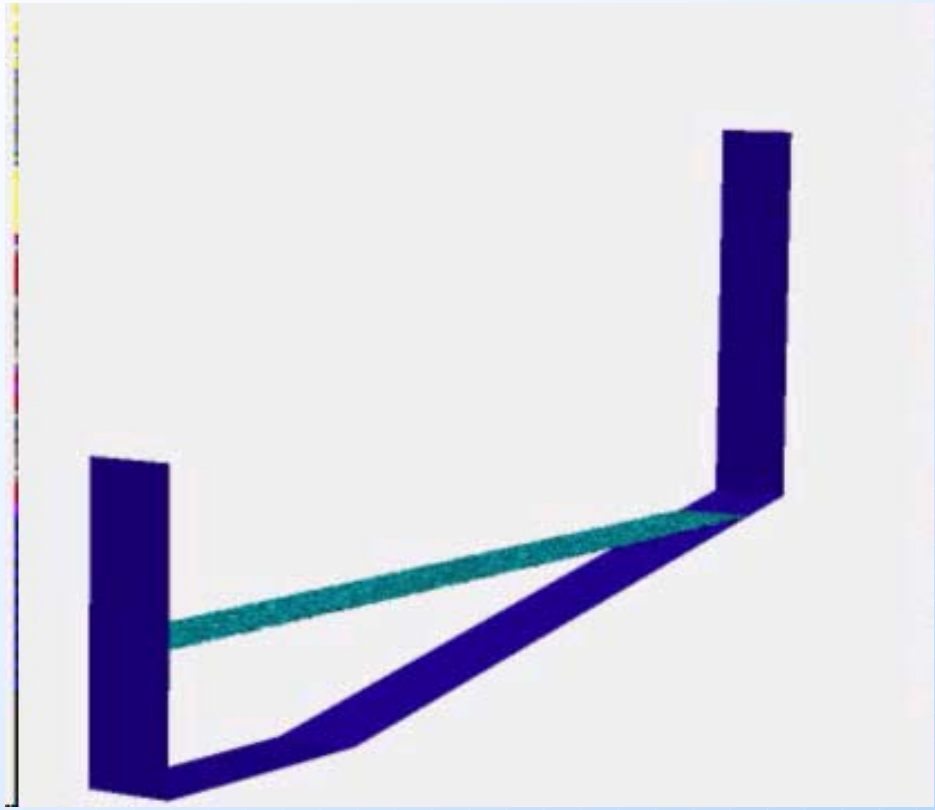
$$\mathbf{M} = \frac{\partial \mathbf{F}}{\partial \mathbf{s}} \qquad \mathbf{N} = \frac{\partial \mathbf{F}}{\partial \mathbf{s}}$$

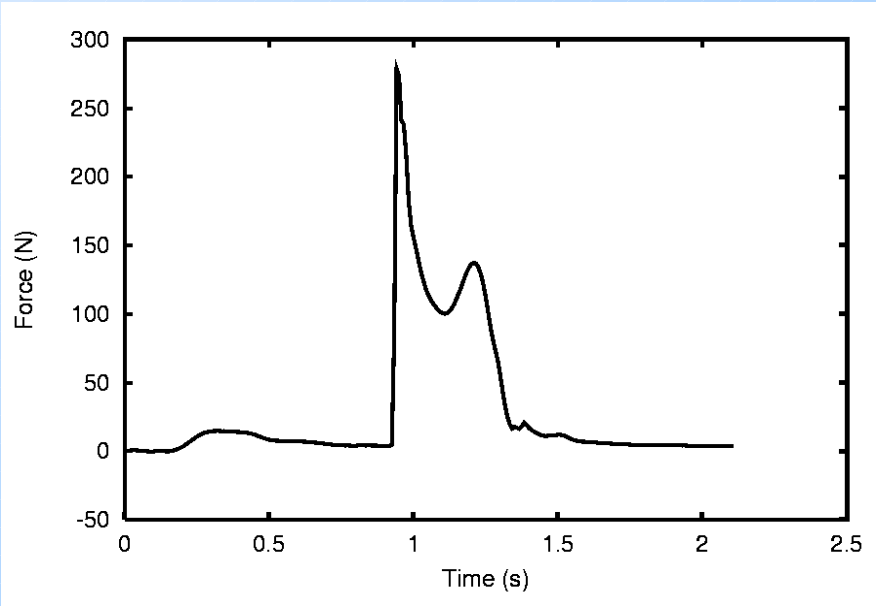
$$\left(\mathbf{M}^{\tau+1} + \frac{\Delta t}{2}\mathbf{N}^{\tau+1}\right)\mathbf{x}^{n+1} = \mathbf{R}^{\tau} \Rightarrow \mathbf{Ax} = \mathbf{b}$$

USING MATRIX-FREE ITERATIVE STRATEGY, WE CAN SOLVE SYSTEM OF EQUATIONS WITH <u>ONE BILLION UNKNOWNS!</u>

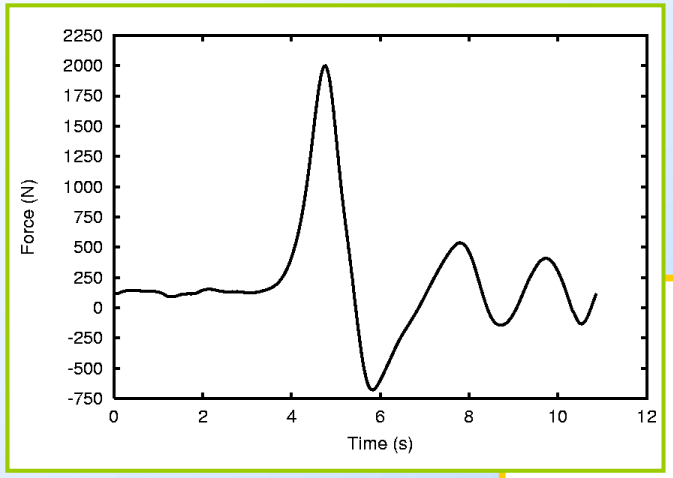
3D Application: Wave Impact

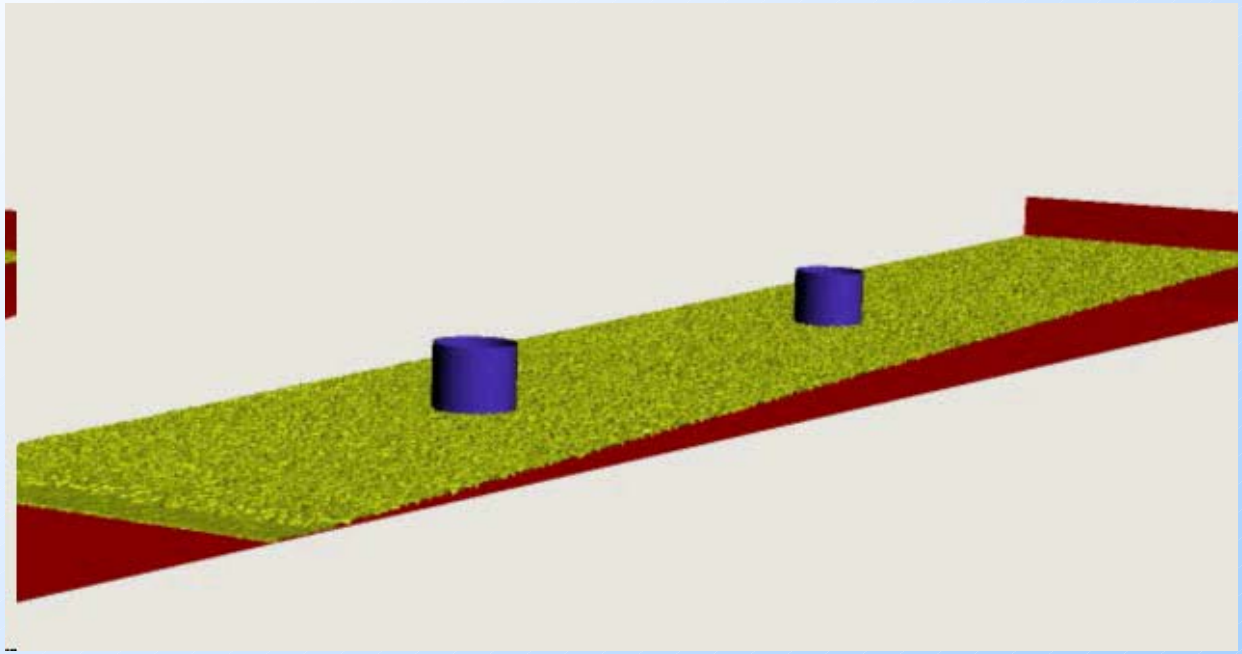


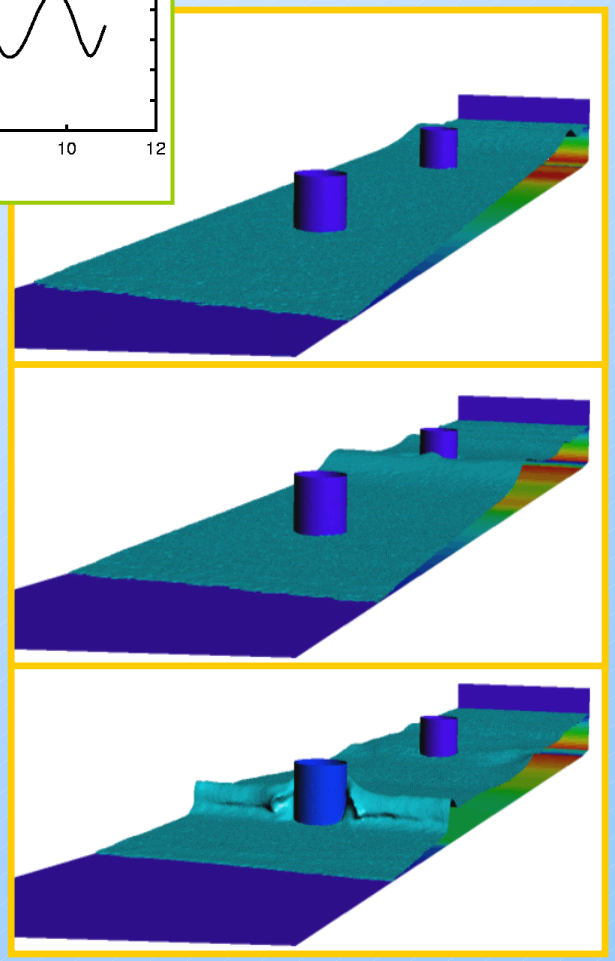




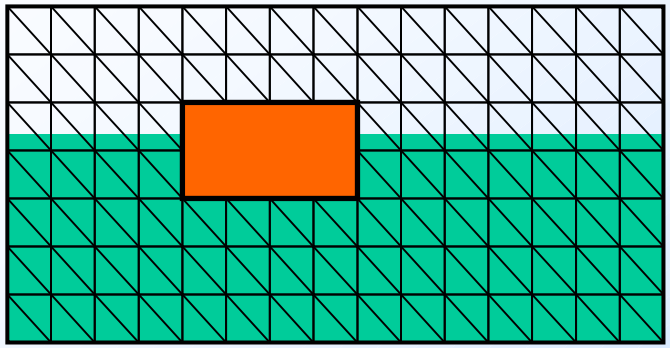
3D Application:
Wave Impact

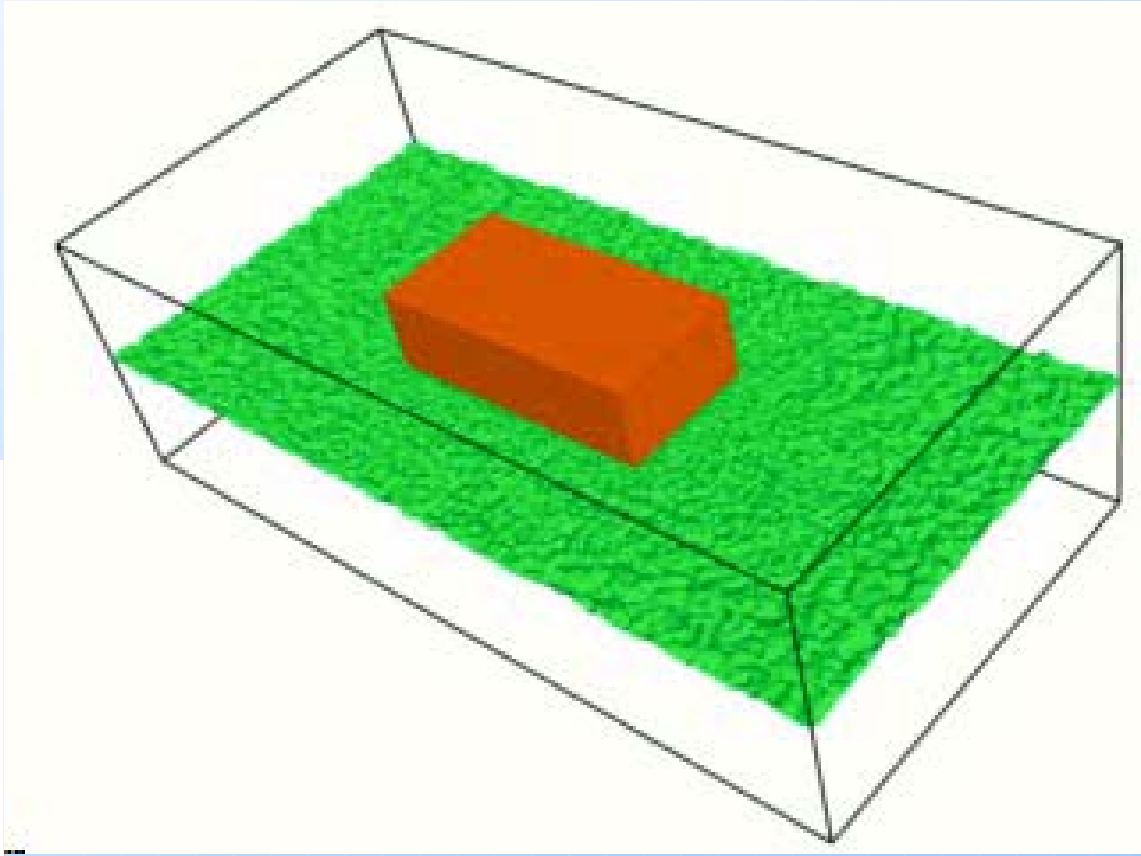


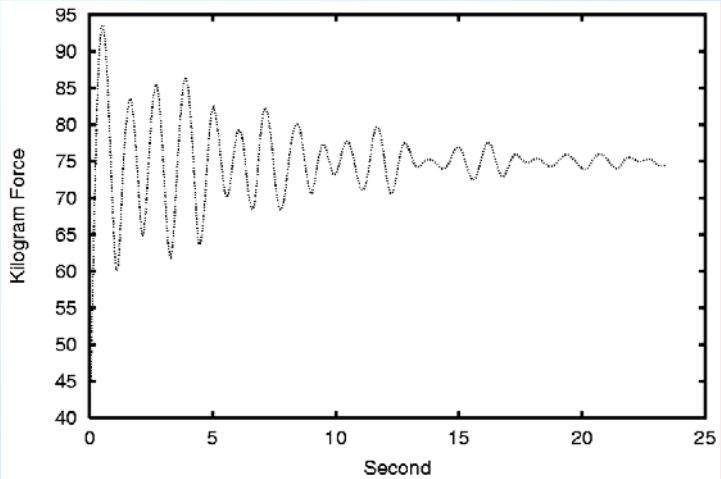




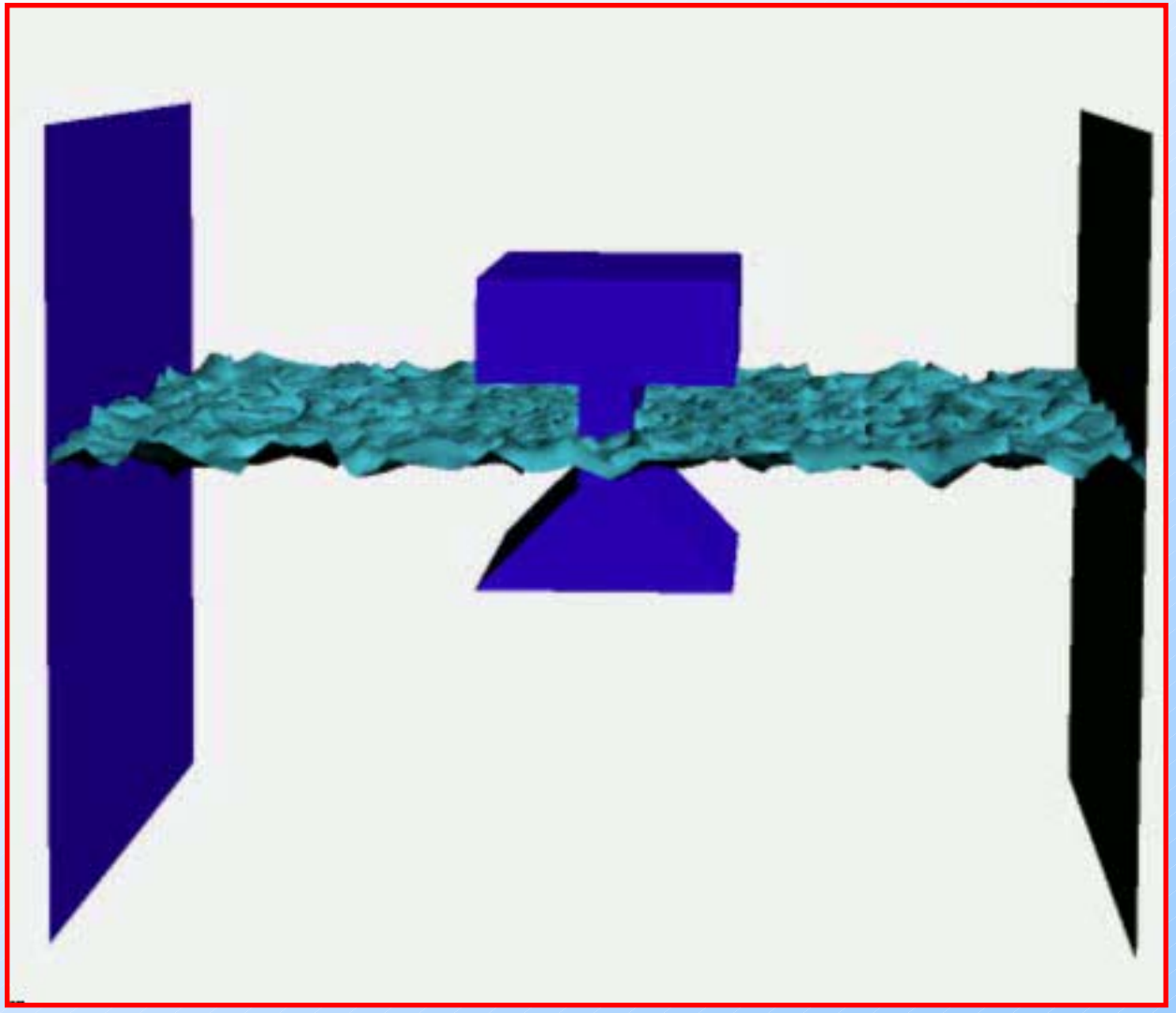
6 DOF Example: Object Dropping in Water





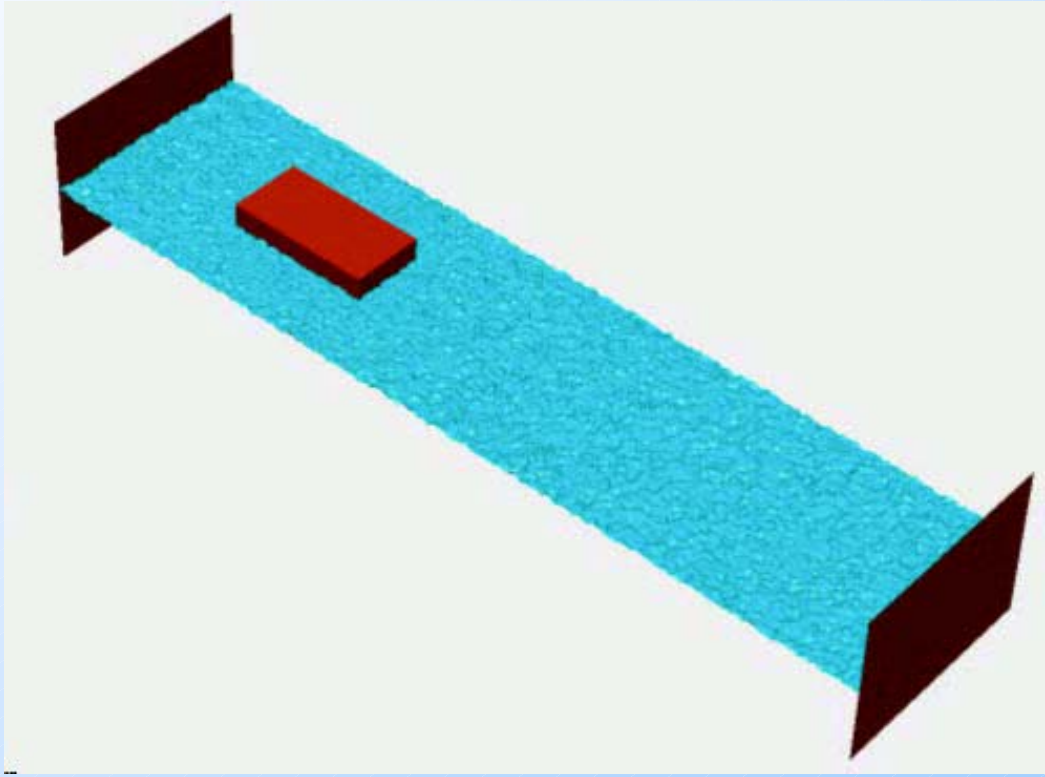


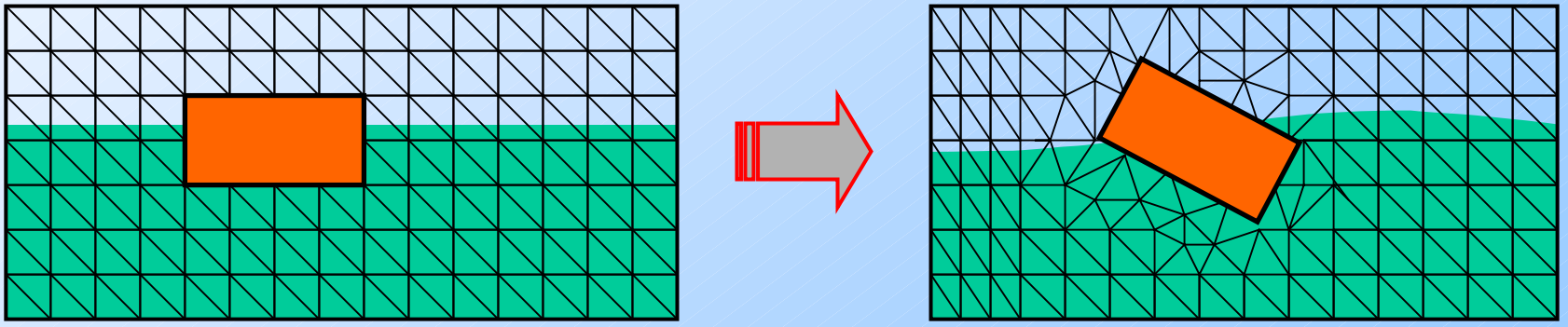
Capability: Floatation and Stability



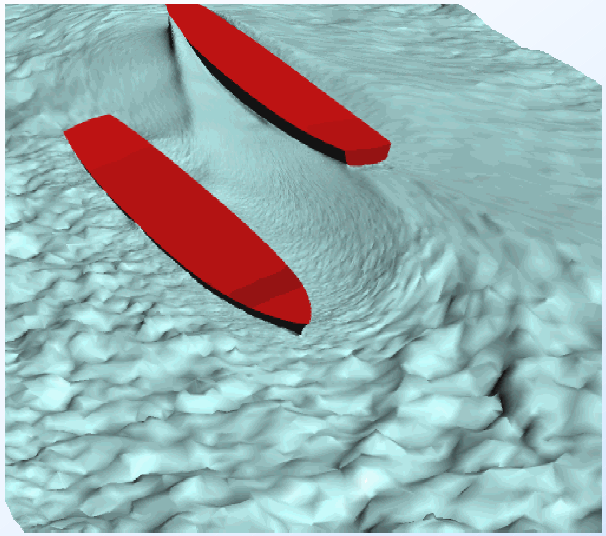
Wave Generator And 6 DOF Example:

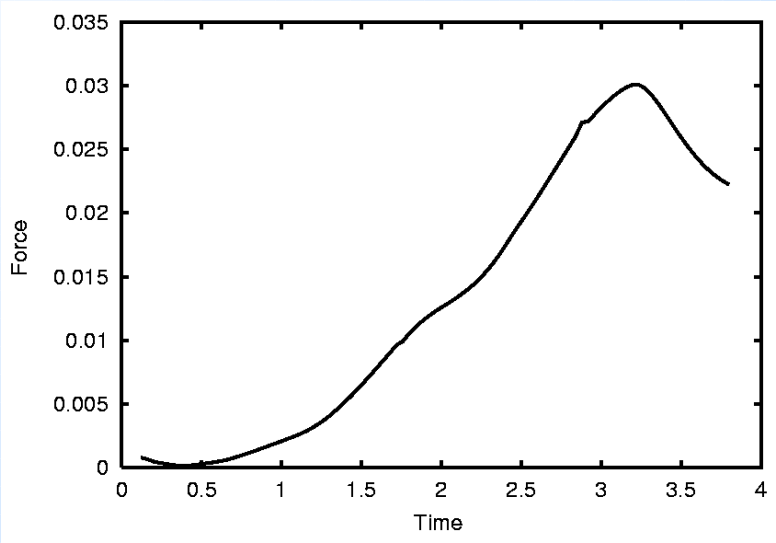
Drifting Object

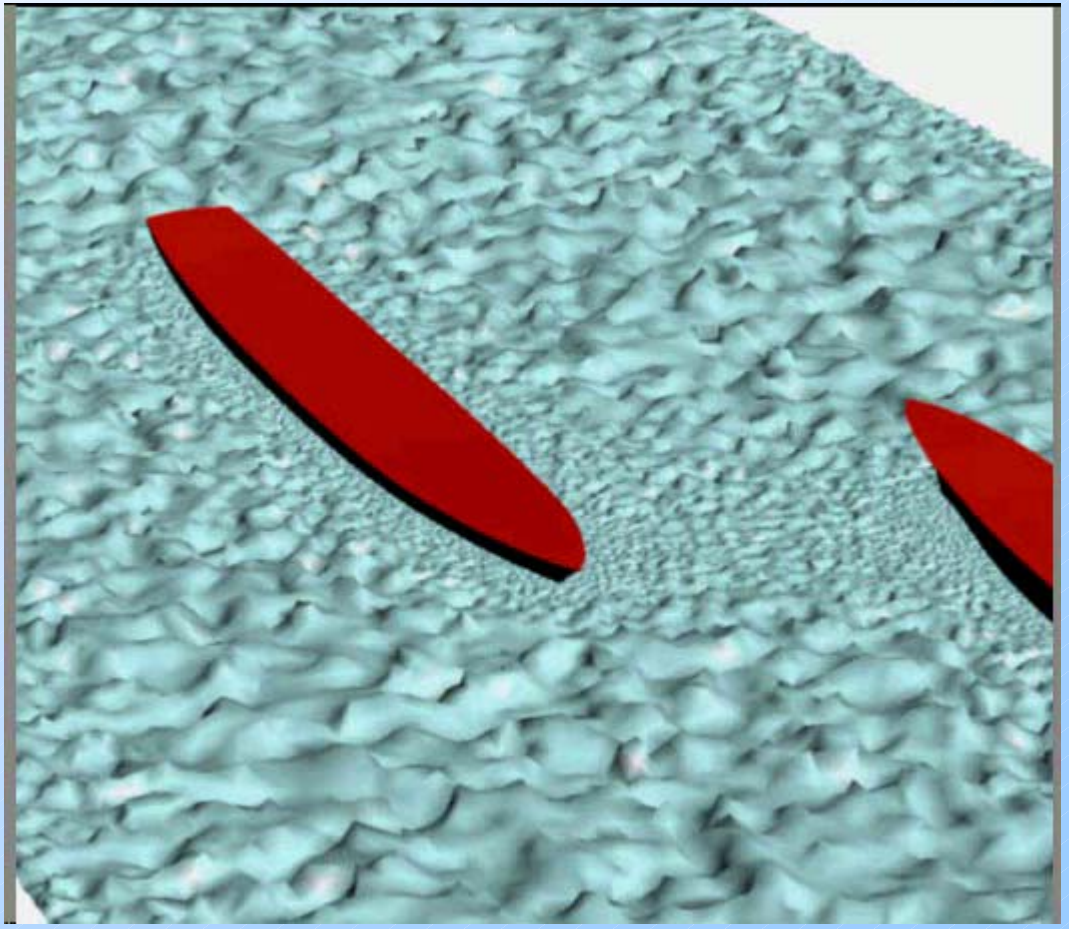




Ship-Ship Interaction





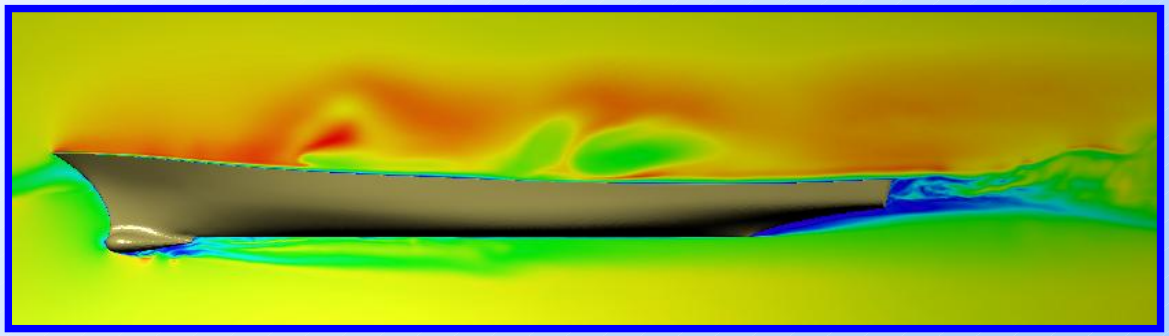


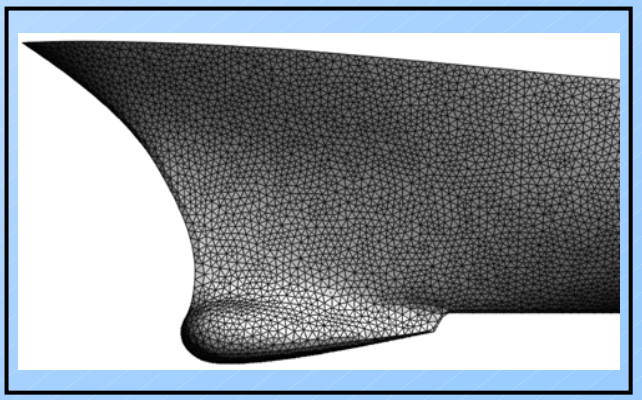
Large-Scale Simulation

Largest Implicit Finite Element Application with Unstructured Mesh

- **One Billion Tetrahedral Elements**
- 875 Million Equations
- 130 Gigaflops Sustained Computation Speed
- Metis Mesh Partitioning
- Parallel Mesh Multiplication
- Remote Visualization

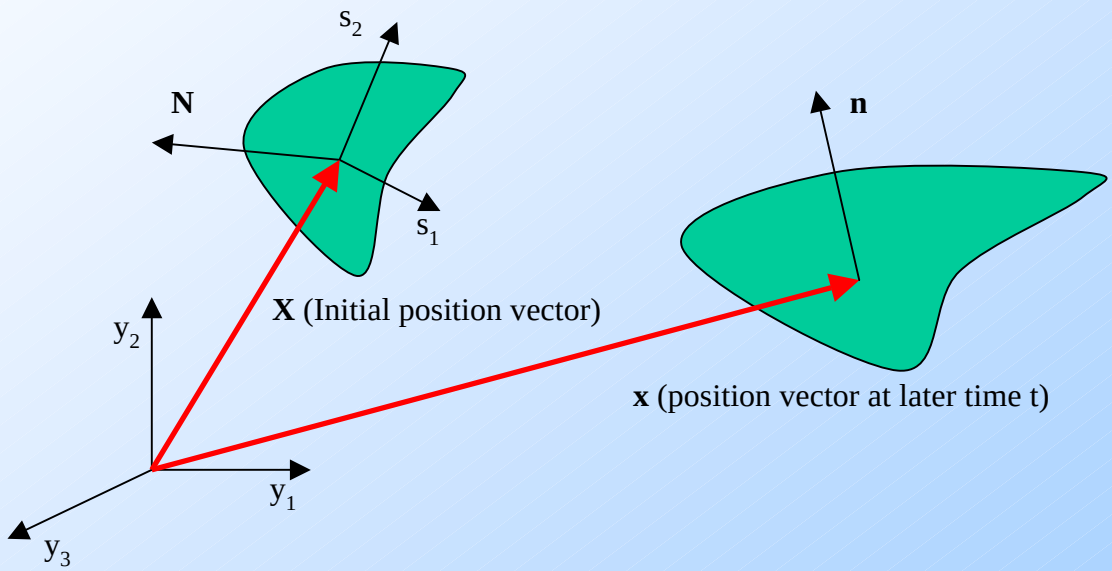
Cray T3E-1200, 1088 Processors,1.3 Teraflops Peak, 557 Gigabytes, World's 7th Fastest Computer





Nonlinear Fabrics and Cables

$$\rho_c\left(\frac{\partial^2 \mathbf{x}}{\partial t^2}-\mathbf{g}\right)=\nabla\cdot\mathbf{T}$$

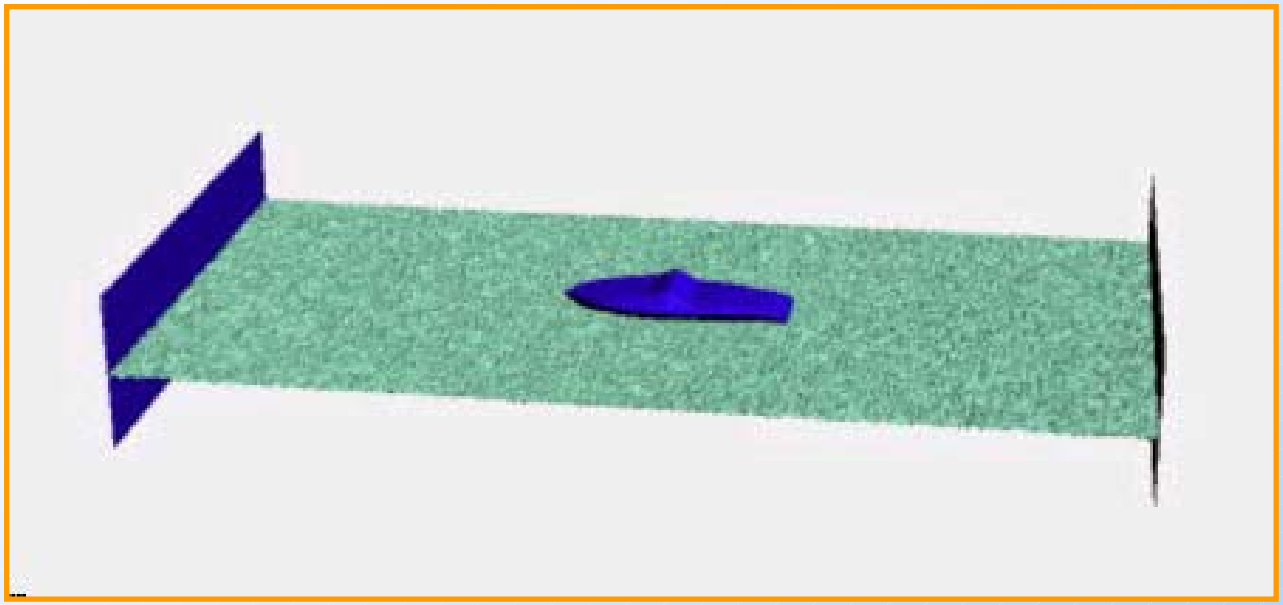


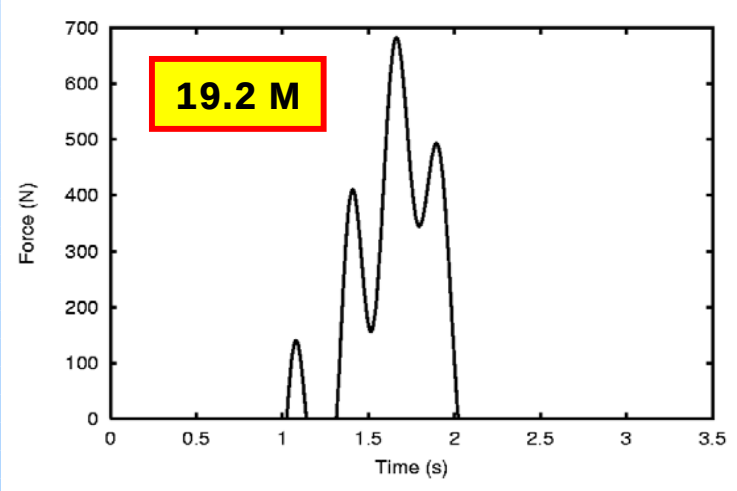
$$\rho_f h_0\int_{\Gamma_0}\mathbf{H}^h\cdot\left(\frac{\partial^2 \mathbf{x}^h}{\partial t^2}-\mathbf{b}\right)d\Gamma+h_0\int_{\Gamma_0}\mathbf{S}_{ij}^{(s)h}:\delta\mathbf{E}_{ij}^{(s)}\,d\Gamma-\int_{\Gamma_0}p^h\,\mathbf{H}^h\cdot\left(\frac{\partial \mathbf{x}^h}{\partial s_1}\times\frac{\partial \mathbf{x}^h}{\partial s_2}\right)d\Gamma=0$$

$$\delta\mathbf{E}_{ij}^{(s)}=\frac{1}{2}\left(\frac{\partial \mathbf{H}}{\partial s_i}\cdot\frac{\partial \mathbf{x}}{\partial s_j}+\frac{\partial \mathbf{x}}{\partial s_i}\cdot\frac{\partial \mathbf{H}}{\partial s_j}\right)$$

Fabrics

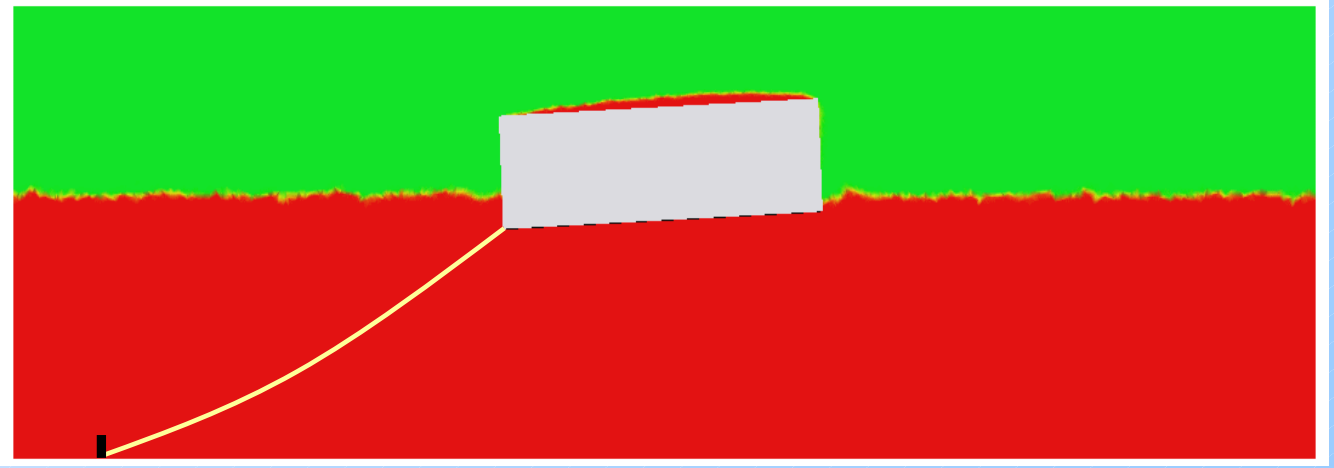
Mooring Force: Floating Boat with a Cable





Free-Surface Flows with Nonlinear Cables

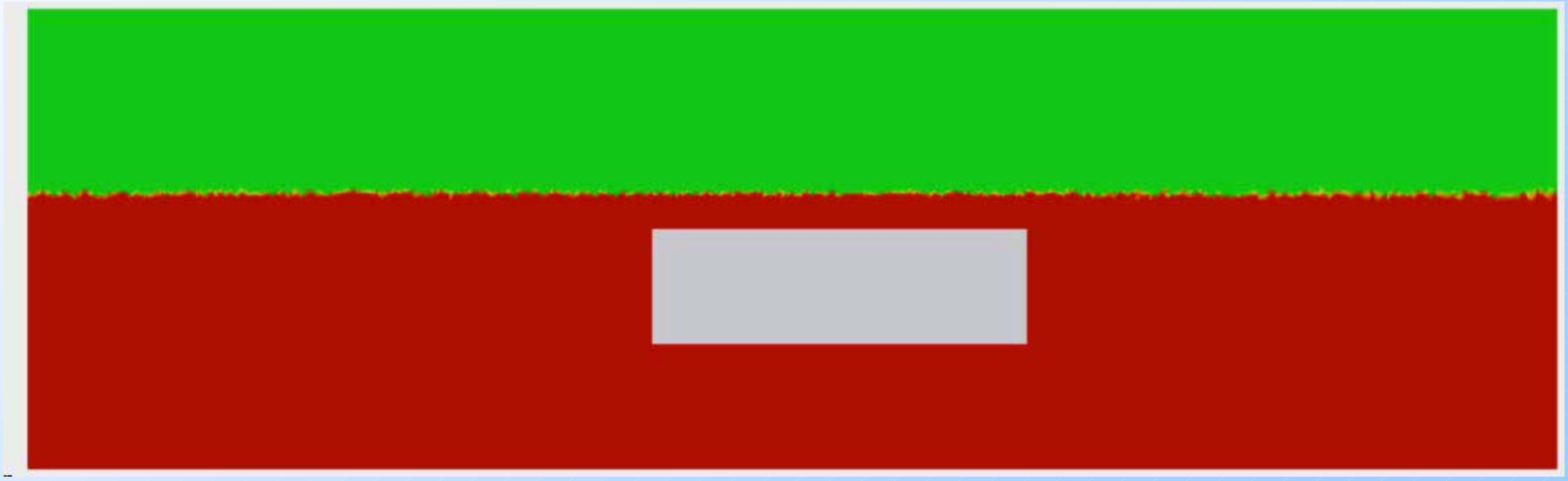
- Density of Cable: 1.5 x Density of Water
- E = 1.0e+3
- A = 1.0e-3



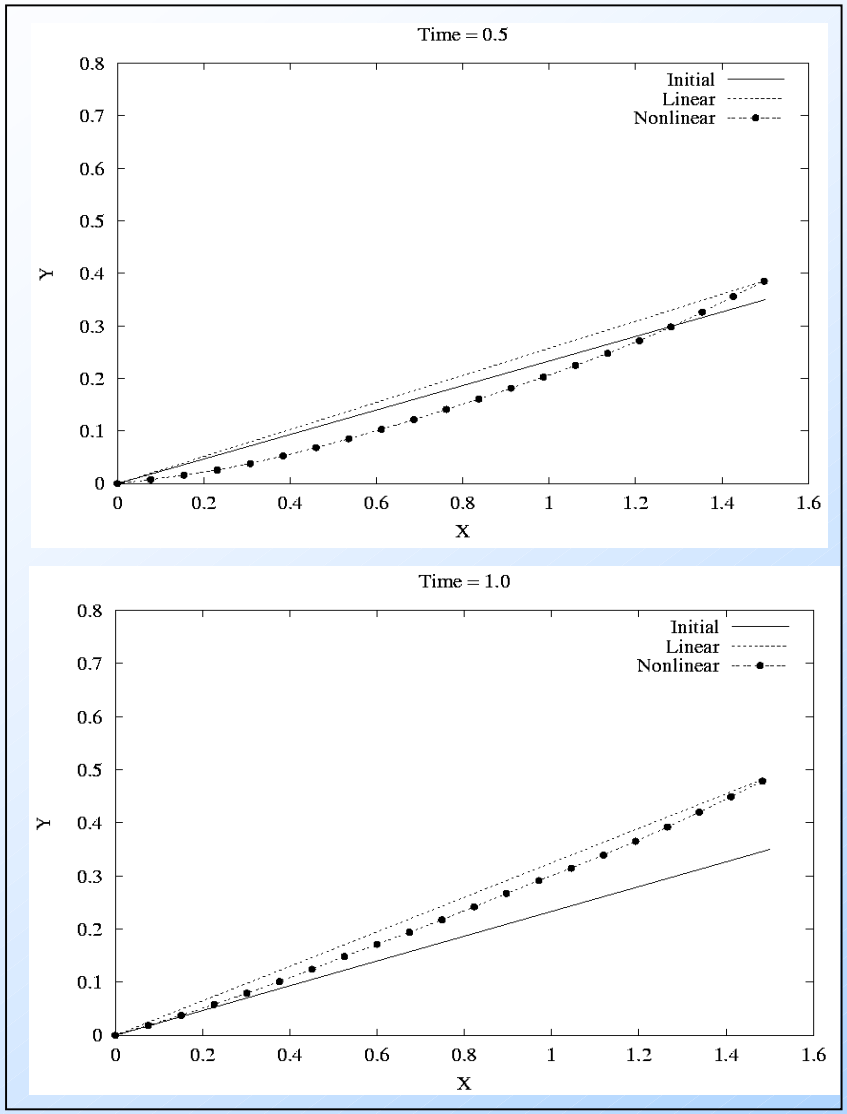
Recent Published Work

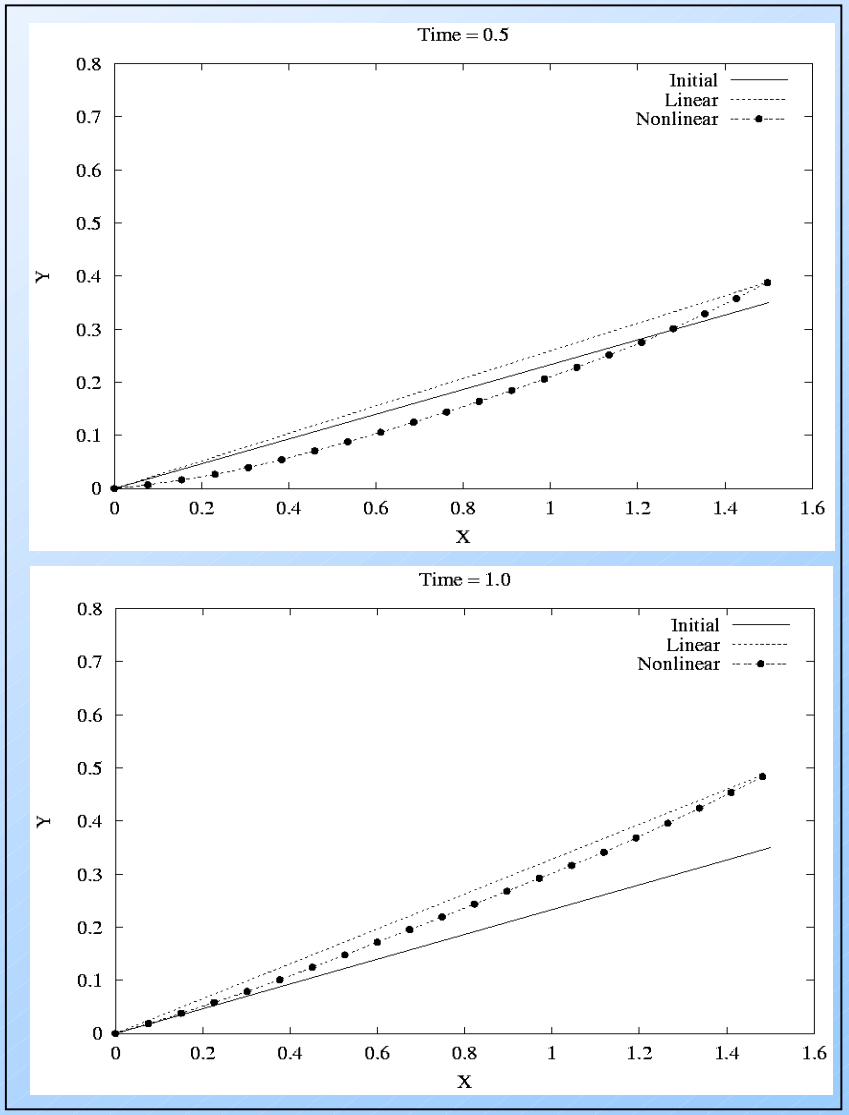
S. Aliabadi, J. Abedi and B. Zellars, "**Parallel Finite Element Simulation of Mooring Forces on Floating Objects**", *International Journal for Numerical Methods in Fluids*, 2003, **41** (8xx-8xx)

- **Coarse Mesh: 1.2 Million Tetrahedral Elements**
- **Fine Mesh: 8 x 1.2 Million Tetrahedral Elements**

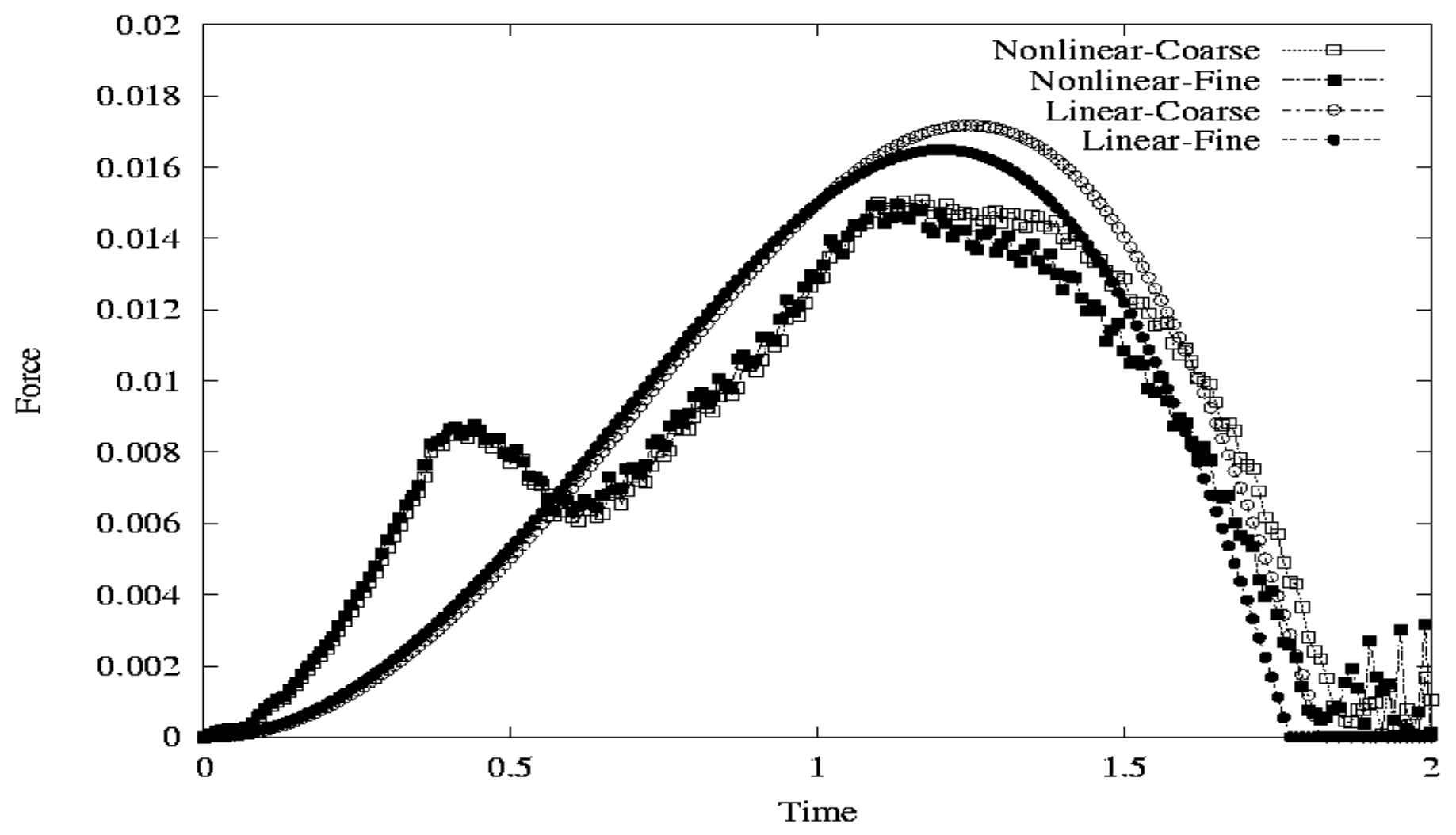


Results


Time = 0.5
Initial
Linear
Nonlinear
Y
X
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Time = 1.0
Initial
Linear
Nonlinear
Y
X


Time = 0.5
Initial
Linear
Nonlinear
Y
X
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Time = 1.0
Initial
Linear
Nonlinear
Y
X

Comparision Between Linear and Nonlinear Cables



Computation on Cray X1

First Production Version of Cray X1
(Serial Number 1)